Stability Analysis and Control of Nonlinear Phenomena in Boost Converters Using Model-Based Takagi–Sugeno Fuzzy Approach

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***Abstract*—The application of a novel Takagi–Sugeno (TS) fuzzy-model-based approach to prohibit the onset of subharmonic instabilities in dc–dc power electronic converters is presented in this paper. The use of a model-based fuzzy approach derived from an average mathematical model to control the nonlinearities in power electronic converters has been reported in the literature, but this is known to act as a low-pass filter, thus ignoring all nonlinear phenomena occurring at converter clock frequency. This paper shows how converter fast-scale instabilities can be captured by extending the TS fuzzy modeling concept to nonsmooth dynamical systems by combining the TS fuzzy modeling technique with nonsmooth Lyapunov stability theory. The new method is applied to the current-mode-controlled boost converter to demonstrate how the stability analysis can be directly applied by formularizing the stability conditions as a numerical problem using linear matrix inequalities. Based on this methodology, a new type of switching fuzzy controller is proposed. The resulting control scheme is able to maintain the stable period-one behavior of the converter over a wide range of operating conditions while improving the transient response of the circuit.**

***Index Terms*—DC–DC converter, linear matrix inequality (LMI), nonsmooth Lyapunov theory, Takagi–Sugeno (TS) fuzzy approach.**

I. Introduction

NONSMOOTH phenomena play an important role in many problems in applied science and engineering. Nonsmooth dynamical systems are modeled by interacting continuous and discrete-event systems. Examples can be found in mechanical systems subjected to unilateral constraints, Coulomb friction or impacts, switching electronic circuits, motion-control systems, computer disk systems, and even enhanced technologies like unmanned aerial vehicles and robotics. In control theory, they frequently appear when discontinuous control laws are involved. Power electronic circuits are considered as archetypal examples for studying the abundance of nonlinear phenomena occurring in this class of systems. From the stability analysis and control point of view, the mathematical study of switching power converters is not easy to tackle, since the resulting models are dynamical systems whose right-hand sides are not continuous or not differentiable. The most widely used approach to study nonlinearities in switching-power-converter circuits is discrete nonlinear modeling [1]–[4], which is mainly focused on examining periodic orbits and their stability instead of the theoretically well-developed stability of equilibria. Nonlinear map-based modeling was proposed by Hamill *et al.*[5] following the early work on sampled-data modeling and is commonly referred to as the Poincaré map method because of the sampling of the state variables at the intersection point of the trajectory with the Poincaré surface [1], [6]. Another method is trajectory sensitivity analysis applied to power converters with the help of a discrete–algebraic–differential model. Trajectory sensitivity analysis examines how the trajectory of the system in state space will respond if the initial condition or other parameters are slightly perturbed [7], [8]. In an alternative approach, Floquet theory has also been used to study the stability of system trajectories by deriving the absolute value of the eigenvalues of the monodromy matrix (i.e., the so-called Floquet multipliers of the system) [9]–[11]. The idea of an auxiliary state vector has also been proposed to simplify the stability analysis when using the well-known Poincaré approach [12]. Although all of the aforementioned methodologies have been successful in providing an insight into fast-scale instabilities of power electronic converters, they cannot be easily applied to design suitable controllers to suppress these nonlinear patterns. The use of the averaged model [1] technique is problematic since it cannot take into account any subharmonic or chaotic behavior that may occur at the switching instants. Different classes of chaos-control strategies based on reliable discrete nonlinear modeling methods [1], [13]–[18] have also been proposed over the years. The implementation of these strategies is still not widely adopted since they are vulnerable to noise and suffer from a high-computation-time requirement [14].

In the search for efficient nonlinear control strategies, the Takagi–Sugeno (TS) fuzzy-model-based control approach integrated with Lyapunov stability analysis has attracted much attention in the past 15 years. Tanaka and Sugeno [19] were the first to formulate the Lyapunov stability condition as a linear matrix inequality (LMI) problem for the stability analysis of TS fuzzy systems. They studied the stability of (discrete-time) fuzzy systems consisting of a weighted sum of linear subsystems resulting from the modeling of a smooth nonlinear dynamical system. The weighting functions were positive scalar functions whose sum was assumed to be equal to unity for all linear subsystems and whose stability is ensured if there exists a common quadratic Lyapunov function for all linear subsystems

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[19]. Since the publication of this seminal paper, there has been considerable amount of research to approximate complicated nonlinear dynamical equations by TS fuzzy systems and formalize the stability analysis of that fuzzy representation based on smooth Lyapunov theory as an LMI problem [20]–[24].

The TS fuzzy approach, either model based or non model based, has already been applied to control power electronic converters [25], [26]. One of the main drawbacks of these previous attempts is the derivation of the fuzzy model from the average dynamical model of the converter, thus ignoring all converter fast-scale instabilities as outlined previously. A TS fuzzy controller based on such a model [25], while demonstrating satisfactory transient performance, cannot predict the nonlinear phenomena occurring at fast timescale [1] and cannot restrain the resulting unstable behavior. On the other hand, applying the non-model-based TS fuzzy control approach [26] is subject to the chronic criticism of black-box design for having no rigorous mathematical stability analysis even if it is able to preserve the nominal period-one operation of the circuit. Addressing this problem, this paper presents a novel and comprehensive analysis to synthesize a TS fuzzy model for an example nonsmooth dynamical system—the current-mode-controlled converter—to thoroughly accommodate the discontinuous switching of the converter. The resulting TS fuzzy model is then able to represent all nonlinear phenomena that take place at clock frequency, including period-doubling bifurcations and chaos. This paper also provides a framework for the stability analysis of the proposed nonsmooth TS fuzzy model of the dc–dc boost converter based on discontinuous Lyapunov functions. The search for Lyapunov functions is formulated as LMIs to make the procedure automatic through interior point methods. Discontinuous Lyapunov functions play a pivotal role in the stability analysis of nonsmooth dynamical systems [27] and the relaxation of the conservative formulation of stability conditions. There are cases in variable-structured systems [28], [29], where continuous Lyapunov functions have been used but only with the skipping of the nonsmoothness of the trajectory. There have also been reports on the use of nonsmooth Lyapunov functions in generic hybrid systems [30]–[32]. However, this paper is the first to apply the nonsmooth Lyapunov approach to the stability analysis of the TS fuzzy representation of a power electronic converter. We demonstrate how the formulation of the infeasibility of stability conditions as LMIs can predict the onset of a period-doubling bifurcation.

The strength of the proposed TS modeling and stability analysis is demonstrated by designing a TS fuzzy switching control strategy to curb the onset of the fast-scale instabilities in the current-mode-controlled boost converter. Based on the developed stability conditions and the fuzzy gain-scheduling concepts, the new controller is designed to achieve the best slow-scale performance while substantially extending the normal period-one operating region of the converter. Stability robustness issues arising from uncertainties in the modeling of switching physical systems are also discussed.

II. Boost Converter And Its Mathematical Model

The current-mode-controlled boost converter circuit (Fig. 1) is a nonsmooth affine system governed by two sets of linear differential equations pertaining to the ON and OFF states of the controlled switch. If the output voltage v_C and the inductor current i are taken as state variables, the state equations during the "ON" period can be defined as

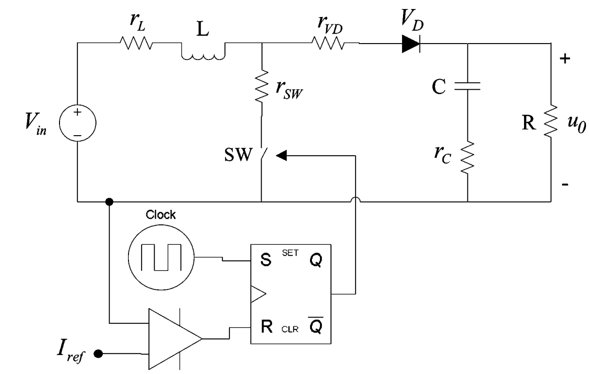

Fig. 1. Boost converter under current-mode control.

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{V_{\text{in}}}{L} - \frac{i}{L}\left(r_L + r_{V_D} + \frac{Rr_C}{R+r_C}\right) - v_C\frac{R}{L(R+r_C)}$$
$$\frac{\mathrm{d}v_C}{\mathrm{d}t} = \frac{1}{C(R+r_C)}(Ri - v_C). \tag{1}$$

and during the "OFF" period, the equations are

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{V_{\text{in}}}{L} - \frac{(r_L + r_{\text{SW}})}{L}i$$
$$\frac{\mathrm{d}v_C}{\mathrm{d}t} = -\frac{v_C}{C(R+r_C)} \tag{2}$$

where r_L, r_C, r_{VD}, and r_{SW} are the parasitic resistances of the inductor, capacitor, diode, and switch, respectively. In a boost converter, the output voltage is always higher than the input voltage. When the controlled switch is turned on, the current through the inductor increases. When the controlled switch is turned off, the polarity of the inductor voltage changes, reducing the inductor current and charging the output capacitor to a voltage higher than the input voltage. We assume continuous conduction mode, where the clock period and the inductor value are such that the inductor current never falls to zero [1].

When switch S is closed, the current through the inductor rises, and any clock pulse arriving during this period is ignored. Switch S opens when the current reaches a reference value I_{ref} and closes upon the arrival of the next clock pulse. The normal operation of the converter referred to period-one operation can be seen in Fig. 2. The control action in the nth clock cycle [26] is given by

$$d(n) = \frac{L}{T(r_L + r_{\text{SW}})} \ln\left(\frac{V_{\text{in}} - (r_L + r_{\text{SW}})i_L(n)}{V_{\text{in}-(r_L+r_{\text{SW}})I_{\text{ref}}}}\right) \tag{3}$$

where $i_L(n)$ denotes the nth sampled input current and the duty cycle $d(n)$ is the ratio between the nth ON time and the clock period T.

Variation in system parameters (for example, input voltage) can lead to border-collision bifurcation and chaos [1], [33], as shown in Fig. 3. In Section III, the converter is modeled by TS fuzzy approach in a way that allows all fast-scale instabilities

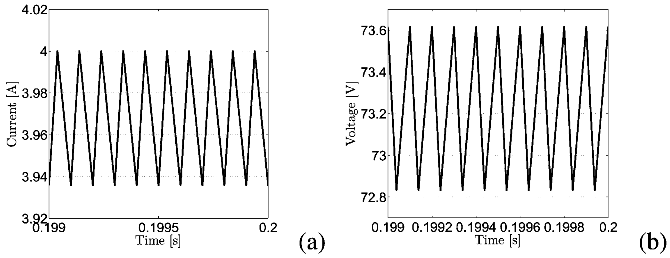

Fig. 2. Nominal period-one operation of the boost converter. (a) Inductor current and (b) output voltage. The nominal values for the parameters are as follows: $V_{\rm in} = 45$ V, $I_{\rm ref} = 4$ A, $L = 27$ mH, $C = 120\ \mu$F, $R = 30\ \Omega$, $r_C = 0.2\ \Omega$, $r_L = 1.2\ \Omega$, $r_{\rm SW} = 0.3\ \Omega$, $r_{\rm VD} = 0.24\ \Omega$, and clock frequency is 10 kHz.

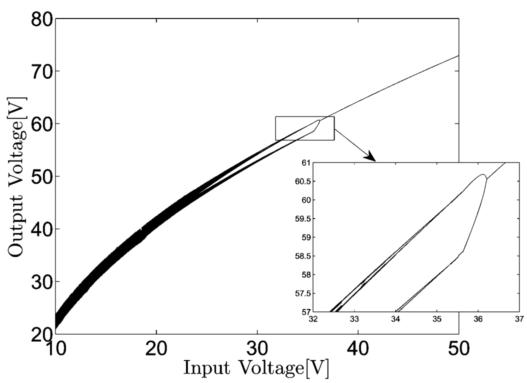

Fig. 3. Bifurcation diagram of the boost converter varying input voltage, using switch-on sampling with conventional peak-current-controlled scheme.

to be accurately studied. For this purpose, the state matrices (1) and (2) are expressed as follows:

$$A_1 = \begin{bmatrix} -\frac{1}{L}(r_L + r_{\rm SW}) & 0 \\ 0 & -\frac{1}{C(R+r_C)} \end{bmatrix} \qquad B_{1,2} = \begin{bmatrix} \frac{V_{\rm in}}{L} \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -\frac{1}{L}\left(r_L + r_{\rm VD} + \frac{Rr_C}{R+r_C}\right) & -\frac{R}{L(R+r_C)} \\ \frac{R}{C(R+r_C)} & -\frac{1}{C(R+r_C)} \end{bmatrix}. \quad (4)$$

III. TS Fuzzy-System Modeling For Analyzing Nonsmooth Dynamical Systems

The fuzzy-system identification and approximation first appeared in the seminal work of Takagi and Sugeno [34] and extended to the application of controller design and stability analysis in [19]. The model is composed of a fuzzy IF–THEN rule base that partitions a space—usually called the universe of discourse—into fuzzy regions described by the rule antecedents. The consequent of each rule j is usually a simple functional expression $(y^j = f^j(x))$. A common format of a rule j is as follows:

$$\text{Rule } j: \quad \text{IF } \theta_1 \text{ is } F_1^j \text{ AND} \ldots \text{AND } \theta_q \text{ is } F_q^j$$
$$\text{THEN } y^j = f^j(x).$$

The vector $\theta \in \Re^q$ contains the premise variables and may be a subset of the independent variables $x \in \Re^n$. Each premise variable θ_i has its own universe of discourse that is partitioned into fuzzy regions by the fuzzy sets describing the linguistic variable F_i^j. The premise variable θ_i belongs to a fuzzy set k with a *truth value* given by the *membership function*$\mu_i^j(\theta_i) : \Re \rightarrow [0,1]$ for $k = 1, 2, \ldots, s_i$, where s_i is the number of fuzzy sets for premise variable i. The notation F_i^j and μ_i^j refers to the linguistic variable and its membership function, respectively, that correspond to the premise variable θ_i in rule j, i.e., $F_{ik}^j \in \left\{F_{i1}^j, F_{i2}^j, \ldots, F_{is_i}^j\right\}$ and $\mu_{ik}^j(\theta_i) \in \left\{\mu_{i1}^j(\theta_i), \mu_{i2}^j(\theta_i), \ldots, \mu_{is_i}^j(\theta_i)\right\}$.

The truth value (or activation degree) h^j for the complete rule j is computed using the aggregation operator AND, also called a *t-norm*, often denoted by $\bigotimes : [0,1] \times [0,1] \rightarrow [0,1]$, $h^j(\theta) = \mu_1^j(\theta_1) \bigotimes \mu_2^j(\theta_2) \bigotimes \cdots \bigotimes \mu_q^j(\theta_q)$. By using a simple algebraic product, the truth value reads $h^j(\theta) = \prod_{i=1}^q \mu_i^j(\theta_i)$. The degree of activation for rule j is then normalized as $w^j(\theta) = h^j(\theta)/\sum_{r=1}^l h^r(\theta)$ where l is the number of rules. This normalization implies that $\sum_{j=1}^l w^j(\theta) = 1$. The response of a TS model, for a given x and θ, is a weighted sum of the consequent functions f^j which reads

$$y = \sum_{j=1}^{l} w^j(\theta) f^j(x).$$

The weighting functions are then denoted as *interpolation functions* because they are used to interpolate local models. It has been shown that a TS fuzzy model of the type

$$\dot{x} = \sum_{j=1}^{l} w^j(\theta)(A^j x + B^j u)$$
$$y = \sum_{j=1}^{l} w^j(\theta) C^j x \quad (5)$$

with the rules and membership functions just explained can approximate any smooth nonlinear function and its first-order derivative [35]. Furthermore, it has been shown [36] that an affine TS system may also be able to approximate the second-order derivatives of a smooth nonlinear function. The boost converter is a *nonsmooth* or *piecewise* smooth dynamical system and can generally be described by an equation of the form:

$$\dot{x} = f(x, \rho) = \begin{cases} f_1(x, u, \rho), & \text{for } x \in \Re_1 \\ f_2(x, u, \rho), & \text{for } x \in \Re_2 \\ \vdots & \\ f_n(x, u, \rho), & \text{for } x \in \Re_n \end{cases} \quad (6)$$

where ρ is a system parameter, u is a continuous input, and $\Re_1$, $\Re_2$, etc., are different regions in the state space, separated by $(n-1)$ dimensional surfaces given by an algebraic equation of the from $\Gamma_n(x) = 0$, called switching manifolds. The application of the fuzzy system modeling of type (5) to the nonsmooth model of the converter calls for modification of the TS fuzzy model (5) which is unable to represent discrete events. Asynchronized discrete-time discrete-state systems or so-called discrete-event systems are commonly described by an equation of the form $m^+(t) = \xi(m(t), \sigma(t))$, where m is the discrete-state variable, σ is the discrete input, and ξ is a function describing the change of m. The input σ takes values in a finite set Σ, and the elements in Σ are commonly called events [37], [38]. In a nonsmooth system (6), both continuous and discrete states influence each other's behavior, a property that a fuzzy model of the form (5) cannot satisfy.

In a rigorous mathematical sense, the fuzzy model (5) can satisfy the local Lipschitz property, which is basically a smoothness requirement since it is implied by continuous differentiability.[1] Nevertheless, systems with discontinuous nonlinearities do not satisfy the Lipschitz property at the points of discontinuity in the sense of the definition of Lipschitz condition [7], [39]. Hence, they demand a fuzzy modeling method that explains the behavior of the system at the points of discontinuity as well as holding the existence (and uniqueness) of the approximative solution. For those reasons, a new fuzzy modeling method is synthesized to represent all the nonlinearities in the converter as follows:

$$\begin{cases} \dot{x} = \sum_{j=1}^{l_m} w^j(\theta, m)(A^j(m)x + B^j(m)u) \\ m^+ = \xi(x, m) \end{cases} \tag{7}$$

where $x \in R^n$ is the continuous state, $m \in M = \{m_1, \dots, m_N\}$ is a discrete state (N possibly infinite), $A^j(m_i) \in \Re^{n\times n}$, $B^j(m_i) \in \Re^n$, $w^j : \Re^n \times M \to [0\ 1]$, $j \in I_{l_m}$, are continuous weighting functions satisfying $\sum_{j=1}^{l_m} w^j(\theta, m) = 1$, and l_m is the number of fuzzy rules. The state space is the Cartesian product $\Re^n \times M$. The function add$\xi : \Re^n \times M \to M$ describes the dynamics of the discrete state. The notation m^+ means the next state of m. The model (7) describes a nonautonomous system in which an external input (for example, the controller clock signal) affects the dynamics of the system.

Each discrete state $m_i \in M$ is associated with a specific fuzzy subsystem $A(m_i)x + B(m_i)u$, $i \in I_N = \{1, 2, \dots, N\}$, or, in general, with a specific set of subsystems $\sum_{j\in\{1,2,\dots\}} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u)$, $i \in I_N = \{1, 2, \dots, N\}$, which we call *subvector field* mainly because it is the fuzzy representation of vector field $f(x, m_i)$ in the fuzzy state space $\mathcal{F}$. Changing the value of m results in a switching to another specific subvector field $\sum_{j\in\{1,2,\dots\}} w^j(x, m_k)(A^j(m_k)x + B^j(m_k)u)$, $k \in I_N$, describing the continuous evolution. The discontinuities in $\dot{x}$ are modeled by subvector-field switching because it implies abrupt changes.

Remark 1: Moving from one discrete state to another can be described by a number of *switch sets*$S_{i,k}$ expressed as

$$S_{i,k} = \{x \in R^n | m_k = \xi(x, m_i)\}, \quad i \in I_N, \ k \in I_N. \tag{8}$$

The switch sets define the change of discrete state m_i to m_k in the continuous fuzzy state space R^n, where $m_i \neq m_k$. The switch sets can often represent hypersurfaces (regions with dim $< n$) □

To ensure the ability of the proposed fuzzy modeling method to represent fast-scale switching events and the ensuing instabilities in the converter, a TS fuzzy model described by (7) and (8) must be obtained. Based on the state-space model of the system (4), the boost converter can be represented by a TS fuzzy model with the following rules.

1) Plant Rule j: IF $x_1(t)$ is F^j THEN

$$\dot{x} = \begin{cases} A^j(m_i)x(t) + B^j(m_i)u(t), & j = 1, 2; i = 1, 2 \\ m^+ = \xi(x, m) \end{cases} \tag{9}$$

where F^j ($j = 1, 2$) are fuzzy sets, $x(t) = [\,i_L(t) \quad v_C(t)\,]^{\mathrm{T}}$, $B^1(m) = B^2(m) = [\,V_{\mathrm{in}}/L \quad 0]^{\mathrm{T}}$, $A^1(m_1) = A^2(m_1) = A^1$ as in (4), $A^1(m_2) = A^2(m_2) = A^2$ as in (4), and two discrete states are defined as $m \in M = \{m_1, m_2\}$. Changing between two discrete states can be expressed by the switch sets

$$\begin{aligned} S_{1,2} &= \{x \in R^n | i_L(dT) - I_{\mathrm{ref}} > 0\} \\ S_{2,1} &= \{x \in R^n | i_L(dT) - I_{\mathrm{ref}} < 0\} \end{aligned} \tag{10}$$

where d is the duty ratio which is calculated from (3) at each clock period and $T = 1 \times 10^{-4}$ s. The membership functions are defined as follows:

$$\begin{aligned} F^1(i_L(t)) &= \frac{1}{2} + \frac{i_L(t) - I_L(0)}{2l} \\ F^2(i_L(t)) &= 1 - F^1(i_L(t)) \end{aligned}$$

where $l = 7.28$ and $I_L(0) = 3.9514$ is selected from the stable fixed point of the original system, which is an intersection point of the limit cycle with the Poincaré map (see [1] and [10] for the detailed calculation using the Newton–Raphson method). Considering $I_L(0)$ in the derivation of the membership functions helps minimize the delay at the switching instances, which may be caused by the fuzzy approximation.

Fig. 4 shows the time responses of the original system under current-mode control and the one modeled using the proposed fuzzy approach under different operating conditions, showing very good agreement.

No delays are introduced by the TS fuzzy model as compared with the original system, as demonstrated by the time responses shown in Fig. 4(a)–(h). The TS fuzzy model also preserves the qualitative behavior of the original system as the supply voltage is varied, as shown by comparing the two bifurcation diagrams in Figs. 3 and 5.

IV. Exponential Stability Analysis

In this section, we present the basic stability conditions for smooth TS systems of the form $\dot{x} = \sum_{i=1}^{l} w^j(\theta)A^j x$ which is assumed to have been obtained through some fuzzy modeling technique [40]. The following theorem states a well-known sufficient condition for the asymptotic stability of polytopic systems [41] of the aforementioned form.

Theorem 1: The system $\dot{x} = \sum_{j=1}^{l} w^j(\theta)A^j x$ is asymptotically stable if there exists a matrix $P = P^{\mathrm{T}}$ such that

$$\begin{cases} P > 0 \\ (A^j)^{\mathrm{T}}P + PA^j < 0 \quad \forall j = 1, 2, \dots, l. \end{cases} \tag{11}$$

The proof can be found in [19] and [41] by considering the smooth quadratic Lyapunov function candidate $V(x) = x^{\mathrm{T}}Px$. From the Rayleigh–Ritz theorem [41], we have

$$\lambda_{\min}(P)\|x\|^2 \leq x^{\mathrm{T}}Px \leq \lambda_{\max}(P)\|x\|^2 \tag{12}$$

[1]We mean locally Lipschitz, if the existence of a unique solution of function f from a given initial point at some initial time t_0 is only guaranteed over an interval $[t_0, t_0 + T]$ where $T > 0$ may be very small [7].

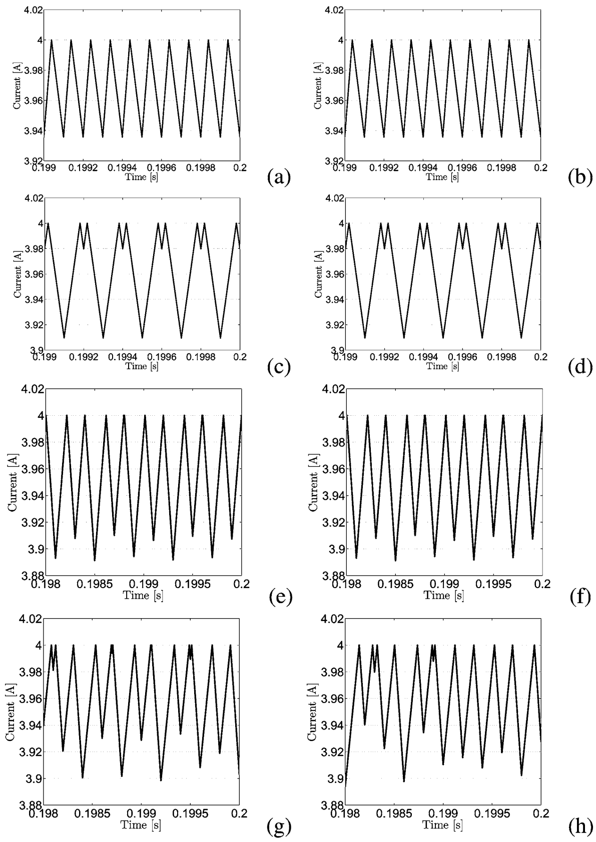

Fig. 4. System responses. (a) Period-one behavior using original system ($V_{\text{in}} = 45$ V). (b) Period-one using proposed TS fuzzy model ($V_{\text{in}} = 45$ V). (c) Period-two behavior using original system ($V_{\text{in}} = 36$ V). (d) Period-two behavior using proposed TS fuzzy model ($V_{\text{in}} = 36$ V). (e) Period-four behavior using original system ($V_{\text{in}} = 34$ V). (f) Period-four behavior using proposed TS fuzzy model ($V_{\text{in}} = 34$ V). (g) Chaotic behavior using original system ($V_{\text{in}} = 20$ V). (h) Chaotic behavior using proposed TS fuzzy model ($V_{\text{in}} = 20$ V).

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues, respectively. Due to the convex combination in $\dot{V}(x)$, we also have

$$\dot{V}(x) \leq -\min_j \lambda_{\max}\left((A^j)^{\mathrm{T}} P + PA^j\right)\|x\|^2. \tag{13}$$

The last two equations fulfill the assumptions of [7, Corollary 3.8], which prove that the fuzzy system is exponentially stable given the condition in Theorem 1. In fact, using a continuous quadratic Lyapunov function is the most widely adopted technique for proving stability for smooth polytopic systems. The reason is that the search for a common positive definite P can be easily automated using efficient convex optimization algorithms for solving systems of LMIs [7], [42]. The purpose of this section is to develop a simple but mathematically rigorous theorem based on nonsmooth Lyapunov functions for the stability analysis of the boost converter represented with TS fuzzy models (9).

The following assumptions of the TS fuzzy system of the general form (7) are made in the stability analysis.

1) It is assumed that each subvector field satisfies a local Lipschitz condition. This means that the local existence of a unique solution is guaranteed for each discrete state [7]. However, global existence of the trajectory can only be guaranteed by some further knowledge of the system behavior, cf. [7].
2) It is assumed without loss of generality that the origin is an equilibrium point to (6). This does not necessarily mean that $f(x, u, \rho) = 0$ for the whole region $\Re$ and, consequently, for all discrete states m_i.

As stated in Section I, existing stability results require the existence of a common quadratic Lyapunov function for all linear subsystems which have to be valid for all states [19], [43]. However, even for smooth dynamical systems represented by TS fuzzy models, finding such a common smooth Lyapunov function is very conservative while the system may be actually stable [44]. To relax the conservativeness, we make it possible to have different representations (7) in the different regions of the fuzzy state space in addition to bringing the nonsmooth Lyapunov functions into play. Therefore, we let the fuzzy state space be partitioned into Δ detached regions $\Omega_q, q \in I_\Delta$ (which means that the partitioning satisfies $\Omega_1 \bigcup \cdots \bigcup \Omega_\Delta = \Omega$ and $\Omega_q \bigcap \Omega_r = \emptyset,\ q \neq r$). It is also assumed that, if the trajectory starts from an initial point in Ω, t_k, $k = 1, 2, \ldots$, it can pass through to another region on the condition that t_k is strictly less than t_{k+1}. Let Λ_{qr} be a set of continuous states for which the trajectory $x(t)$, with initial states $(x_0, m_0) \in \mathcal{F}_0$ (initial fuzzy states will be denoted by $\mathcal{F}_0$ where $\mathcal{F}_0 \subseteq \mathcal{F}$ consists of all initial combinations of the states that may occur) pass from Ω_q to Ω_r, i.e.,

$$\Lambda_{qr} = \{x \in \Re^n | \exists t < t_0, \text{ such that } x(t^-) \in \Omega_q, x(t) \in \Omega_r\}. \tag{14}$$

Note that it is necessary that Ω_q and Ω_r are neighboring sets if $\Lambda_{qr} \neq \emptyset$. Furthermore, Λ_{qr} is given by hypersurfaces. However, this is not sufficient, since the trajectory must also pass from one region to another. Let

$$I_\Lambda = \{(q, r) | \Lambda_{qr} \neq \emptyset\} \tag{15}$$

which is a set of tuples indicating that there is at least one point for which the trajectory passes from Ω_q to Ω_r. Let us define $V_q : \mathbf{cl}\Omega_q^x \to \Re,\ q \in I_\Delta$, as a (scalar) continuous function which represents the system's (abstract) energy in region Ω_q (**cl** denotes the closure of a set, which is the smallest closed set containing the set). Hence, the overall energy can be defined as

$$V(x) = V_q(x) \text{ when } (x, m) \in \Omega_q \tag{16}$$

where $V(x)$ is a discontinuous Lyapunov function at the neighboring regions $\Lambda_{qr}, (q, r) \in I_\Lambda$. Considering the assumption of $t_k < t_{k+1}$ for every trajectory with an initial point in Ω, $V(x)$ is *piecewise continuous* as a function of time. As $V_q(x)$ is assumed to be continuously differentiable on $\mathbf{cl}\Omega_q^x$, $q \in I_\Delta$. Using (7), the time derivative of $V_q(x)$ is

$$\dot{V}_q(x) = \sum_{j=1}^{l_m} w^j(\theta, m) \frac{\partial V_q(x)}{\partial t}\left(A^j(m)x + B^j(m)\right), \quad (x, m) \in \Omega_q \tag{17}$$

which depends on the discrete state m.

To end up with an LMI problem, the candidate Lyapunov function will be piecewise quadratic, meaning that each local Lyapunov function has the structure

$$(x,m)\in\Omega_q, \qquad V_q(x)=\pi_q+2p_q^{\mathrm{T}}x+x^{\mathrm{T}}P_qx \tag{18}$$

where $\pi_q \in \Re$, $p_q \in \Re^n$, and $P_q = P_q^{\mathrm{T}} \in \Re^n \times \Re^n$, $q \in I_\Delta$. By defining

$$\tilde{x}=\begin{bmatrix} x \\ 1\end{bmatrix} \quad \tilde{P}_q=\begin{bmatrix} P_q & p_q \\ p_q^{\mathrm{T}} & \pi_q\end{bmatrix} \tag{19}$$

$V_q(x)$ can be written as

$$(x,m)\in\Omega_q, \qquad V_q(x)=\tilde{x}^{\mathrm{T}}\tilde{P}_q\tilde{x}. \tag{20}$$

Now, by defining $\tilde{I}=\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix}$ where $\tilde{I}\in\Re^{n+1}\times\Re^{n+1}$ and $\tilde{A}(m_i)=\begin{bmatrix} A(m_i) & B(m_i) \\ 0 & 0\end{bmatrix}$, $m_i \in M$, we state the stability theorem as follows.

Theorem 2: The fixed point is exponentially stable in the sense of Lyapunov, if there exist $\tilde{P}_q$, $q \in I_\Delta$, and constants $\alpha > 0$, $\beta > 0$, and $\gamma > 0$, such that the following are satisfied:

1) $x \in \Omega_q^x$, $\alpha\tilde{x}^{\mathrm{T}}\tilde{I}\tilde{x} \le \tilde{x}^{\mathrm{T}}\tilde{P}_q\tilde{x} \le \beta\tilde{x}^{\mathrm{T}}\tilde{I}\tilde{x}$, $q \in I_\Delta$.
2) $(x,m) \in \Omega_q$, $\tilde{x}^{\mathrm{T}}(\tilde{A}(m)^{\mathrm{T}}\tilde{P}_q + \tilde{P}_q\tilde{A}(m))\tilde{x} \le -\gamma\tilde{x}^{\mathrm{T}}\tilde{I}\tilde{x}$, $m_i \in \Omega_q^m$, $q \in I_\Delta$.
3) $x \in \Lambda_{qr}^x$, $\tilde{x}^{\mathrm{T}}\tilde{P}_r\tilde{x} \le \tilde{x}^{\mathrm{T}}\tilde{P}_q\tilde{x}$, $(q,r) \in I_\Lambda$.

The proof of this theorem can be found in the Appendix.

Remark 2: Note that, in the case of a nonunique definition of the next discrete state, the conditions for the energy V_q to be nonincreasing must be valid for all possible subvector-field switches. For instance, if the discrete state m is changed on a surface from m_i to m_k or m_z, the energy has to decrease for both these possibilities. □

The aforementioned stability conditions are confined to be satisfied in a part of continuous state space. The first condition is restricted to the region Ω_q^x, the second to the region Ω_q^{x,m_i}, and the third to Λ_{qr}^x. It is possible, by expressing the regions by positive (quadratic) functions and employing an $\mathcal{S}$-procedure [41] technique, to substitute the confined conditions with unconfined conditions. In the following analysis, this procedure is first explained in general terms and then applied to the confined conditions in the stability theorem.

Let $Q_0,\ldots,Q_s$ be quadratic functions of the variable $x \in \Re^n$ of the form:

$$Q_k(x)=x^{\mathrm{T}}Z_kx+2c_k^{\mathrm{T}}x+d_k, \qquad k=0,\ldots,s \tag{21}$$

where $Z_k = Z_k^{\mathrm{T}}$. Consider the following condition:

$$Q_0(x)\ge 0 \text{ in the region } \{x\in\Re^n | F_k(x)\ge 0, k\in I_s\}. \tag{22}$$

In our case, the first two conditions in the stability theorems are of the form

$$Q_0(x)\ge 0 \text{ in the region } \Omega \tag{23}$$

where Ω is a region Ω_q^x and $Q_0(x) \ge 0$ is the corresponding condition in the region. By finding quadratic functions $Q_0(x) \ge 0$, $k \in I_s$ such that

$$\Omega \in \{x\in\Re^n | Q_k(x)\ge 0, k\in I_s\}.$$

Clearly, if (22) is satisfied, so is (23). The extreme case is to let $Q_0(x) \ge 0$ in the entire state space; nevertheless, specifying a larger than necessary $\{x \in \Re^n | Q_k(x) \ge 0, k \in I_s\}$ should be avoided, since this conservatism may result in not finding the solution for the original condition (23). By including Ω in a region specified by quadratic functions, we can replace the confined condition (22) by an unconfined condition as follows.

Lemma [41]: If there exist $\delta_k \ge 0$, $k \in I_s$, such that

$$\forall\, x\in\Re^n, \qquad Q_0(x)\ge\sum_{k=1}^{s}\delta_kQ_k(x) \tag{24}$$

then (22) holds. Hence, by introducing additional variables $\delta_k \ge 0$, $k \in I_s$, condition (23) has been transformed into an LMI as

$$\tilde{x}^{\mathrm{T}}\begin{bmatrix} Z_0 & c_0 \\ c_0^{\mathrm{T}} & d_0\end{bmatrix}\tilde{x}\ge\sum_{k=1}^{s}\delta_k\tilde{x}^{\mathrm{T}}\begin{bmatrix} Z_k & c_k \\ c_k^{\mathrm{T}} & d_k\end{bmatrix}\tilde{x}. \tag{25}$$

The replacement of (22) by the **Lemma** may be conservative. However, it can be shown that the converse is true in case of single quadratic form, i.e., $s = 1$[41]. Applying the stability conditions, the region partitioning Ω_q, $q \in I_\Delta$, should be represented by a single quadratic form. Then, (22), (23), and the Lemma are equivalent. As the regions Λ_{qr}, $(q,r) \in I_\Lambda$ are given by hypersurfaces (region with dim $< n$), they can be defined by $Q_k(x) = 0$, $k \in I_s$, where $Q_k(x)$ has the form (21). In this case, there is no requirement for the different δ_k, $k \in I_s$, to be greater or equal to zero in the Lemma, since the Lemma holds despite the sign of these constants. Nonetheless, if some switch surface cannot be exactly described by $Q_k(x) = 0$, $k \in I_s$, then it is possible to conservatively describe such a switch surface by larger switch regions, cf. the discussion after (23). Now, by assuming that Ω_q, where $q \in I_\Delta$, are given by $Q_k^q(x) \ge 0$, $k \in I_{s_q}$, and the regions Λ_{qr}, where $(q,r) \in I_\Lambda$, are given by $Q_k^{qr}(x) \ge 0$, $k \in I_{s_{qr}}$, and by substituting the conditions given in the different regions by the condition on the form of (25), the LMI problem is reformulated as follows.

1) LMI Problem: If there exist $\tilde{P}_q$, where $q \in I_\Delta$, and constants $\alpha, \beta > 0$, $\mu_k^q \ge 0$, ν_k^{qij}, and η_k^{qr} and if there is a solution to min β subject to

$$\begin{bmatrix} \alpha & 0 \\ 0 & 0\end{bmatrix}+\sum_{k=1}^{s_q}\mu_k^q\begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^{\mathrm{T}} & d_k^q\end{bmatrix}\le\tilde{P}_q$$

$$\tilde{P}_q\le\begin{bmatrix} \beta & 0 \\ 0 & 0\end{bmatrix}+\sum_{k=1}^{s_q}\mu_k^q\begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^{\mathrm{T}} & d_k^q\end{bmatrix}, \qquad q\in I_\Delta$$

$$(q,i,j)\in I_\Omega,\ \begin{bmatrix} A_j & B_j \\ 0 & 0\end{bmatrix}^{\mathrm{T}}\tilde{P}_q+\tilde{P}_q\begin{bmatrix} A_j & B_j \\ 0 & 0\end{bmatrix} +\sum_{k=1}^{s_{qij}}\nu_k^{qij}\begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^{\mathrm{T}} & d_k^q\end{bmatrix}\le 0, \qquad q\in I_\Delta$$

$$\tilde{P}_r\le\tilde{P}_q-\sum_{k=1}^{s_{qr}}\eta_k^{qr}\begin{bmatrix} Z_k^{qr} & c_k^{qr} \\ (c_k^{qr})^{\mathrm{T}} & d_k^{qr}\end{bmatrix}, \qquad (q,r)\in I_\Lambda \tag{26}$$

then the fixed point is exponentially stable in the sense of Lyapunov.

To ascertain the applicability of the aforementioned LMI problem in studying the fast-scale instabilities of the converter, we recall the fuzzy model of the converter (9) operating under conventional current-mode control and set the operating point $(V_{\rm in}, I_{\rm ref}) = (36.24\ \text{V}, 4\ \text{A})$, which we already know is a stable period-one operating point (Fig. 3). In addition, the fuzzy state space is portioned into two regions, as stated in the following with the S-procedure technique explained earlier:

$$\begin{aligned}\Omega_1 &= \{(x,m) \in H | x \in \Re^n, m = m_1\}\\ \Omega_2 &= \{(x,m) \in H | x \in \Re^n, m = m_2\}.\end{aligned} \tag{27}$$

Solving the LMI problem results in a solution

$$\tilde{\mathbf{P}}_1 = \begin{bmatrix} 1.2397 & 0 & 0.5956 \\ 0 & 1.1200 & 0 \\ 0.5956 & 0 & 2.4795 \end{bmatrix} \tag{28}$$

$$\tilde{\mathbf{P}}_2 = \begin{bmatrix} 129.3565 & -2.2224 & 86.9494 \\ -2.2224 & 1.0404 & -1.7740 \\ 86.9494 & -1.7740 & 96.9417 \end{bmatrix} \tag{29}$$

with the optimal value of $\beta = 166.9391$. Hence, the system is globally exponentially stable. Solving the LMI problem for $V_{\rm in} = 36.10$ V, the operating point where period-one passes through period-two (Fig. 3), results in an infeasible solution. This means that the LMI formulation previously mentioned can accurately detect the occurrence of bifurcation phenomena in the boost converter, which, otherwise, is only achievable by employing complicated discrete nonlinear modeling.

Note that it is not possible to find a common $\tilde{P}$ such that the stability conditions are satisfied. Moreover, by defining the fuzzy state space in one region instead of two, the LMI problem is found infeasible even with discontinuous-Lyapunov-function-based analysis for the stable period-one operating point of $(V_{\rm in}, I_{\rm ref}) = (36.24\ \text{V}, 4\ \text{A})$; thus, the region partitioning is essential to eliminate the conservative formulation of the stability conditions of Theorem 2.

V. Controller Design

This section is concerned with the design of a new TS fuzzy controller that extends the nominal period-one behavior of the circuit to a large operating domain and, at the same time, boosts the slow-scale performance of the converter. For this purpose, we should involve the nonautonomous fuzzy model of the converter (7) in the design process. The controller design has two objectives. First, it is assumed that a number of different subvector fields are available as a result of discrete states (or function) so that a number of different local controllers need to be designed for every subvector field in every region $\tilde{\Omega}_i$. It is also assumed that there is an associated local Lyapunov function for every subvector field $\sum_{j\in\{1,2,\ldots\}} w^j(x,m)(A^j(m)x + B^j(m)u)$ for every region $\tilde{\Omega}_i$. Such local Lyapunov functions may naturally exist when the subvector fields are a result of local control-law design based on the specified local Lyapunov functions. The second objective is to decide the location of the switch sets (or function) such that the first and third LMI conditions of the LMI problem outlined in Section IV along with the LMI condition of Theorem 3, presented later in this section, are fulfilled, guaranteeing the stability of the closed-loop fuzzy system. It is certainly not hard to motivate the need for such design techniques in practical cases. From a more general point of view, there are many controller structures in industry consisting of locally designed controllers and logic deciding the switching strategy among these [45]. The local controllers may be designed by linear feedback theory but will be only valid in certain operating regions of the state space due to nonlinearities in the system. Controller structures consisting of a linear regulator whose parameters are changed as a function of the operating conditions in a predetermined way are called gain schedulers [46]. To design the local controllers, the concept of gain schedulers is employed; thus, the jth rule of the control input can be defined as follows.

1) Control Rule j: IF θ_1 is F_1^j and … and θ_q is F_q^j THEN

$$\begin{cases} u(t) = -K^j(m)x(t), & j = 1,\ldots,l_m \\ m^+ = \xi(x,m). \end{cases} \tag{30}$$

A local state-feedback controller is designed for each subvector field. The fuzzy controller is inferred as follows:

$$\begin{cases} u(t) = \sum_{j=1}^{l_m} w^j(\theta,m)K^j(m)x(t) \\ m^+ = \xi(x,m). \end{cases} \tag{31}$$

By substituting (31) in (7), the closed-loop system can be represented as (32), shown at the bottom of the page. The first part of the fuzzy regulator design is to determine the local feedback gains $K^j(m)$. The region in the continuous state space where the subvector field $\sum_{j\in\{1,2,\ldots\}} w^j(x,m)(A^j(m)x + B^j(m)u)$ is allowed to be selected is specified by a set of continuous states denoted by $\tilde{\Omega}$. Define $F(x,m)$ as shown at the bottom of the page, which denotes the set of all subvector fields to be selected for the continuous state x. It is assumed that the set $F(x,m)$ is not empty at every state in the region $\tilde{\Omega}$ of validity.

$$\begin{cases} \dot{x} = \sum_{j=1}^{l_m} \sum_{i=1}^{l_m} w^j(\theta,m)w^i(\theta,m)(A^j(m)x + B^j(m)K^i(m)))x(t) \\ m^+ = \xi(x,m). \end{cases} \tag{32}$$

$$F(x,m) = \left\{ \sum_{j\in\{1,2,\ldots\}} A^j(m_i)x + B^j(m_i) \Big| \sum_{j\in\{1,2,\ldots\}} A^j(m_i)x + B^j(m_i) \text{ is allowed to be selected in } \tilde{\Omega}_i \right\}$$

By this assumption, at least some subvector fields can be selected at each continuous state x. This can be expressed by the covering condition

$$\bigcup_{i=1}^{N} \tilde{\Omega}_i = \tilde{\Omega}.$$

Furthermore, it is assumed that there exist at least two overlapping regions $\tilde{\Omega}_i$ and $\tilde{\Omega}_j, i \neq j$, where the values of the subvector fields differ (to have a nontrivial design problem). The regions $\tilde{\Omega}_i$ already cover the state space, implying that the first condition of stability in Theorem 2, and accordingly in the LMI problem, is satisfied. The second condition of stability has to be reformulated with regard to the LMI-based design to determine local fuzzy regulator feedback gains $K^j(m)$.

Theorem 3: Let $X \in \Re^n \times \Re^n$ be a diagonal positive definite matrix. The system (7) can be exponentially stabilized via the fuzzy controller (31) in the region $x \in \Omega_q^{x,m_i}$, where the second condition of Theorem 2 is replaced by

$$\begin{aligned} x \in \Omega_q^{x,m_i}, \\ \tilde{x}^{\mathrm{T}}((\bar{A}^j(m)^{\mathrm{T}} + \bar{G}^{ji}(m)^{\mathrm{T}})\tilde{P}_q + \tilde{P}_q(\bar{A}^j(m) \\ + \bar{G}^{ji}(m)) + \bar{X}(m)\tilde{P}_q\bar{X}(m))\tilde{x} < 0, \\ m_i \in \Omega_q^m,\ q \in I_\Delta \end{aligned} \quad (33)$$

where $\bar{A}^j(m) = \begin{bmatrix} A^j(m) & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{G}^{ij}(m) = \begin{bmatrix} -B^j(m)K^i(m) & 0 \\ 0 & 0 \end{bmatrix}$, and $X(m) = \begin{bmatrix} X(m) & 0 \\ 0 & 0 \end{bmatrix}$.

Proof: Choose the discontinuous quadratic Lyapunov function of the form defined in (18) and (19). For the vector field in (32), the time derivative of $V_q(x)$ according to (17) can be written as

$$\dot{V}_q(x) = \sum_{j=1}^{l_m}\sum_{i=1}^{l_m} w^j(\theta,m)w^i(\theta,m)\frac{\partial V_q(x)}{\partial x}(A^j(m)x + B^j(m)K^i(m))x. \quad (34)$$

It follows directly from (34) that, if

$$\frac{\partial V_q(x)}{\partial x}\left(A^j(m)x + B^j(m)K^i(m)\right)x \leq 0, \quad x \in \Omega^{x,m_i,j};\ m_i \in \Omega_q^m$$

then $\dot{V}_q(x) \leq 0$, $q \in I_\Delta$, due to the fact that $w^i w^j \geq 0$. Using a quadratic Lyapunov function (18), this condition can be formulated as

$$\begin{aligned} &\dot{x}^{\mathrm{T}}P_q x + \dot{x}^{\mathrm{T}}p_q + xP_q\dot{x} + p_q^{\mathrm{T}}\dot{x} \\ &= \tilde{x}^{\mathrm{T}}(\bar{A}^j(m)^{\mathrm{T}}\tilde{P}_q + \tilde{P}_q\bar{A}^j(m) + \bar{G}^{ji}(m)^{\mathrm{T}}\tilde{P}_q + \bar{G}^{ji}(m)\tilde{P}_q)\tilde{x} \end{aligned}$$

where $\bar{A}_j(m) = \begin{bmatrix} A^j(m) & 0 \\ 0 & 0 \end{bmatrix}$ and $\bar{G}^{ij}(m) = \begin{bmatrix} -B^j(m)K^i(m) & 0 \\ 0 & 0 \end{bmatrix}$.

Let $X(m)$ be a diagonal positive definite matrix so we can write

$$\begin{aligned} \tilde{x}^{\mathrm{T}}((\bar{A}^j(m)^{\mathrm{T}} + \bar{G}^{ji}(m)^{\mathrm{T}})\tilde{P}_q + \tilde{P}_q(\bar{A}^j(m) + \bar{G}^{ji}(m)) \\ + \bar{X}(m)\tilde{P}_q\bar{X}(m))\tilde{x} < 0, \qquad \text{for all } i \in I_{l_m};\ j \in I_{l_m}. \end{aligned} \quad (35)$$

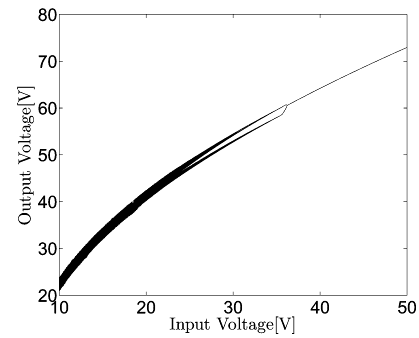

Fig. 5. Bifurcation diagram of the boost converter varying input voltage. Switch-on sampling with the synthesized TS fuzzy model.

From (35), it follows that

$$\dot{V}_q\left(x(t)\right) \leq \dot{\tilde{x}}^T(t)\bar{X}(m)\tilde{P}_q\bar{X}(m)\tilde{x}(t) \leq -\beta V\left(x(t)\right)$$

where $\beta = -(\lambda_{\min}(\bar{X}(m)\tilde{P}_q\bar{X}(m))/\lambda_{\max}(P_q))$ and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimal and maximal eigenvalues of the matrix, respectively. Hence

$$V\left(\tilde{x}(t)\right) \leq V(0)\mathrm{e}^{-\beta t}.$$

Therefore, $\|\ \tilde{x}(t)\ \|^2 \leq (V(0)/\lambda_{\min}(\tilde{P}_q))\mathrm{e}^{-\beta t}$ is concluded. □

Remark 3: Similar to the approach proposed in [25] for a smooth system, the diagonal matrix $X(m)$ has been introduced to achieve a faster decay rate for the trajectory associated with each vector field m_i and, hence, better transient performance for each controller gain $K^i(m)$ associated with each subvector field. Choosing the right matrix $X(m_i)$ can cause the switching system to converge faster to the fixed point if the system satisfies the stability condition in Theorem 2. Therefore, exponential stability can coexist with best transient performance. □

Fulfilling the first and second conditions of stability according to Theorem 2, it remains only to position the switch sets (if possible) such that the energy decreases at each discrete state associated with a local Lyapunov function according to the third condition of Theorem 2. The proposed method to design the switch sets follows as a result of the stability conditions stated in Theorems 2 and 3. As mentioned earlier, naturally local Lyapunov functions are available from the local control-law design based on Theorem 3. If the location of switch sets can be designed such that Theorems 2 and 3 are satisfied, then the stability of the closed-loop fuzzy system is guaranteed. The following simple example illustrates this idea.

2) Example: Assume that the two quadratic Lyapunov functions are available, described by matrices

$$P_1 = \begin{bmatrix} 1 & -5 \\ -5 & 50 \end{bmatrix} \quad P_2 = \begin{bmatrix} 50 & 5 \\ 5 & 1 \end{bmatrix}$$

which stabilize the corresponding fuzzy systems with linear subsystems

$$A(m_1) = \begin{bmatrix} 1 & 3 \\ 0 & 0.1 \end{bmatrix} \quad A(m_2) = \begin{bmatrix} 0.1 & 0 \\ -3 & 1 \end{bmatrix}$$

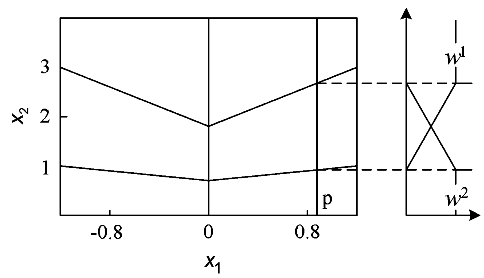

Fig. 6. Hatched regions are states satisfying (left) $x^{\mathrm{T}}(A(m_1)P_1 + P_1A(m_1))x \leq 0$, (middle) $x^{\mathrm{T}}(A(m_2)P_2 + P_2A(m_2))x \leq 0$, and (right) $x^{\mathrm{T}}(P_2 - P_1)x \leq 0$.

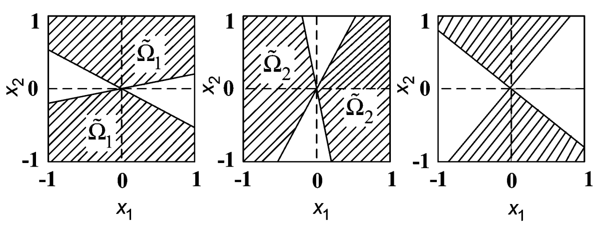

Fig. 7. Weighting function $w^j(\theta)$ for a specific value p of x_1.

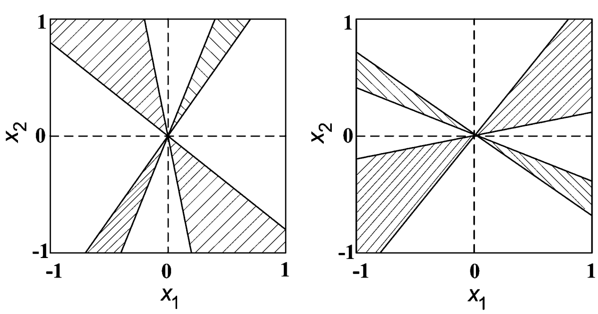

Fig. 8. Hatched regions mark states where both local Lyapunov functions decrease and where the energy decreases in the case of switching from $A(m_1)x$ to $A(m_2)x$ and from $A(m_2)x$ to $A(m_1)x$.

in the shaded regions shown in the left and middle photographs in Fig. 6. These two regions cover the state space an overlap each other at certain states where a specific subvector field has to be chosen from the two possible fields.

The weighting functions for this fuzzy system are shown in Fig. 7. The vertices of the weighting functions for a specific value p of x_1 are given by the intersection of the vertical $x_1 = p$ and the hyperplanes in Fig. 7. The regions where both local Lyapunov functions are decreasing are shown in Fig. 8. With the knowledge of the vector-field direction, it can be concluded that the stability conditions are satisfied if the vector fields are selected such that $A(m_1)$ is switched to $A(m_2)$ somewhere in the selected region in the first and third quadrants of Fig. 8 (left) and is not changed unless the states in the second and fourth quadrants of Fig. 8 (right) are reached, in which case $A(m_2)$ is switched to $A(m_1)$ and so on. The vector-field switchings may occur anywhere in the shaded regions, and one possible choice is shown in Fig. 9 together with a trajectory simulation.

By switching in the interior of the shaded region, the global exponential stability conditions in Theorem 2 are satisfied.

In the approach proposed here, it is possible to change subvector fields and the corresponding Lyapunov functions at every state where the overall energy decreases, whether equal or not. The goal is to locate the switch sets such that the stated stability conditions are satisfied. However, the assumption that every change of subvector field should occur when local Lyapunov functions are equal leads, in some cases, to an unstable system. Selecting the subvector field corresponding to the smallest Lyapunov function results in the trajectory reaching the boundary of the region where the energy is guaranteed to decrease, measured by the local Lyapunov function. Hence, there must be a switch to the other subvector field but this forced change will not occur at states where the energies of the two local Lyapunov functions are equal, as required, for instance, by the assumptions made in [47]. The practical consequence of the switching strategy proposed in this section is greater design flexibility, as illustrated in Example 2. The best design for the switch sets depends on the given Lyapunov functions. When these are obtained by designing local controllers as proposed by Theorem 3, the possibility of success in satisfying the stability conditions increases with a larger number of local controllers overlapping each other in large regions. Nevertheless, it is always possible to succeed in the design of switch sets whenever there is at least one subvector field that is stable in the entire region of validity (with an associated Lyapunov function).

To design the fuzzy controller (31), first, we set the entries of the diagonal matrices $X(m_1) = \mathrm{diag}([9.6 \quad 16 \quad 0])$ and $X(m_2) = \mathrm{diag}([8 \quad 12.5 \quad 0])$ and maintain the regions as defined in (27). Then, the control gains are obtained by solving the LMI problem in Theorem 3 and the remaining conditions in LMI problem 1 as follows:

$$\begin{aligned} K^1(m_1) &= K^2(m_1) = [3.6198, 3.5370] \\ K^1(m_2) &= K^2(m_2) = [3.5037, 0.1179]. \end{aligned} \tag{36}$$

Therefore, the control action $u(t)$ can be given as (37), shown at the bottom of the page. Moreover, regarding (3), the duty ratio can be set to

$$d(t) = \frac{L}{T(r_L + r_{\mathrm{SW}})} \ln\left(\frac{V_{\mathrm{in}} - (r_L + r_{SW})u(t)}{V_{\mathrm{in}} - (r_L + r_{\mathrm{SW}})I_{\mathrm{ref}}}\right). \tag{38}$$

After designing the local controllers, what remains is to locate the switch set to satisfy the third stability condition of Theorem 2. For this purpose, we define acceptable switch

$$\begin{cases} u(t) = -F^1(i_L)(K_1^1(m_1)i_L + K_2^1(m_1)v_c) - F^2(i_L)(K_1^1(m_2)i_L + K_2^1(m_2)v_c) \\ m^+ = \xi(x, m). \end{cases} \tag{37}$$

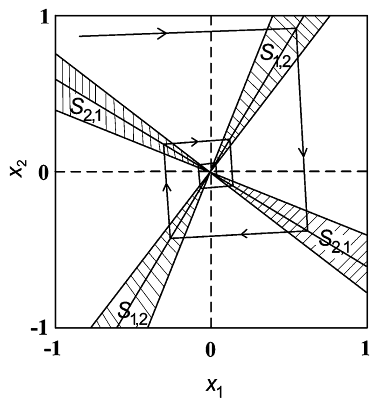

Fig. 9. Switching anywhere in the hatched regions results in a stable closed-loop system if the subvector-field switchings occur in the interior of the shaded regions.

regions that can fulfill the third stability condition in Theorem 2. The acceptable switch regions for the converter are given by

$$\begin{aligned}\bar{S}_{1,2} &= \{x \in \Re^n | x^{\mathrm{T}}(\tilde{P}_2 - \tilde{P}_1)x \leq 0\} \\ &= \{x \in \Re^n | 225.84x_1^2(dT) + 10.24x_1(dT)x_2(dT) \\ &\qquad - 330.63x_2^2(dT) \leq 39.68\} \\ \bar{S}_{2,1} &= \{x \in \Re^n | x^{\mathrm{T}}(\tilde{P}_2 - \tilde{P}_1)x \geq 0\} \\ &= \{x \in \Re^n | 225.84x_1^2(dT) + 10.24x_1(dT)x_2(dT) \\ &\qquad - 330.63x_2^2(dT) \geq 39.68\}.\end{aligned}$$

Locating the switch set in any area within the acceptable switch regions (39) guarantees that the energy (local Lyapunov functions) decreases at every switching region and, therefore, the exponential stability of the closed-loop fuzzy system. The justification of the acceptable switch regions is further discussed in the robustness analysis presented in Section VI.

In order to illustrate the major improvement in converter behavior, when operating under the proposed TS fuzzy control scheme, both in terms of its fast-scale and slow-scale performances, a comparison with the converter performance when operating under its conventional control scheme (Fig. 1) is made. Fig. 10(a) shows how the original control scheme is unable to regulate the system response when operating at $(V_{\rm in}, I_{\rm ref}) = (20, 4)$. The proposed TS fuzzy control scheme can bring the plant to a stable period-one region and ensure the best regulation, as shown in Fig. 10(b).

Comparing the bifurcation diagrams of the two systems, the proposed TS fuzzy control scheme can clearly preserve the stable period-one behavior of the circuit over a much larger operating range. The proposed TS fuzzy controller successfully eliminated the nonlinear phenomena identified (Fig. 11(a)) and extended the period-one behavior (Fig. 11(b)) to a much broader region of reference current values compared with the black-box-designed TS fuzzy controller proposed by Guesmi *et al.* [26]. The exceptional performance of the proposed approach is also shown in Fig. 12, where the stable period-one operating region is extended for a significantly wider range of input voltage values. To check the performance of the proposed control approach in terms of the transient response of the

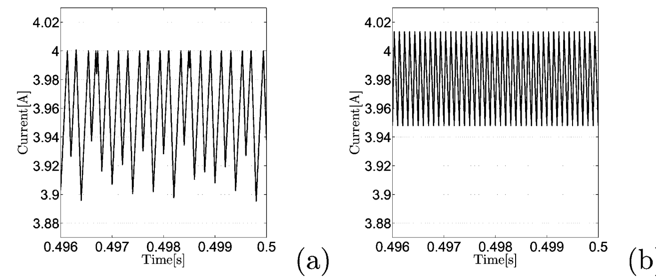

Fig. 10. System responses is (a) chaotic under the conventional control scheme and (b) regulated under the proposed TS fuzzy control scheme.

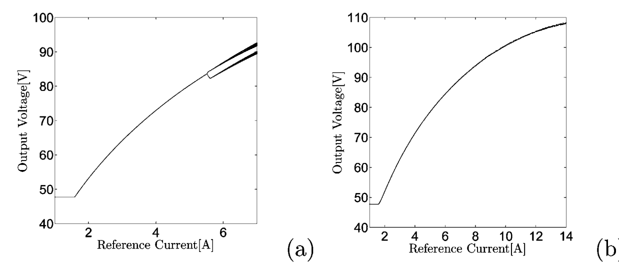

Fig. 11. Bifurcation diagram varying reference current with (a) the conventional control scheme and (b) the proposed TS fuzzy control scheme.

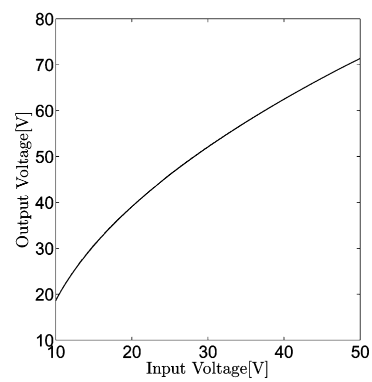

Fig. 12. Bifurcation diagram varying supply voltage with the proposed TS fuzzy control scheme.

system, abrupt variations of supply voltage, reference current, and load were carried out. Fig. 13(a) and (b) shows the superior transient behavior of the TS fuzzy controller when the supply voltage is changed from 45 to 85 V at 0.1 s and back to 45 V at 0.2 s. A similar improvement in performance can be seen when a large step change in reference current is made from 4 to 6 A, causing a period-doubling bifurcation [Fig. 14(b)] in the circuit using the original control scheme but maintaining the stable period-one operation of the circuit when operating under the proposed TS fuzzy controller [Fig. 14(a)]. A sudden step change in load from 30 to 50 Ω and back to 30 Ω at 0.1 and 0.2 s, respectively, also causes instability when operating under the original control scheme, as shown in Fig. 15(a), whereas stability is maintained when using the proposed TS fuzzy control method [Fig. 15(b)].

VI. Robustness Analysis

If the stability of a fuzzy system of the type (7) with nominal given switch sets is shown by the Lyapunov theory, the nominal

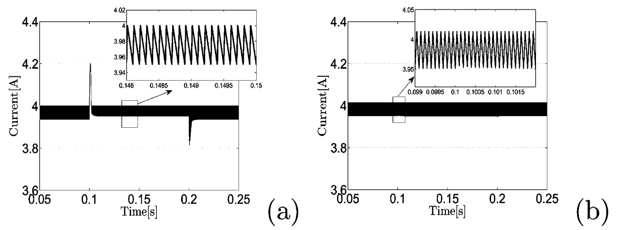

Fig. 13. Output current response of the converter subject to sudden supply voltage changing from 45 to 85 V and from 65 to 45 V at 0.1 and 0.2 s, respectively, with $R = 30\ \Omega$ under the (a) conventional control scheme and (b) proposed TS fuzzy control scheme.

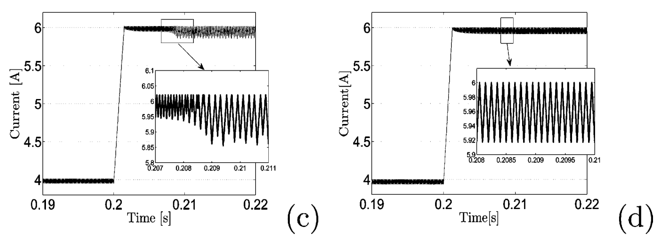

Fig. 14. Output current response of the converter subject to large reference current step changing from 4 to 6 A at 0.2 s with $V_{\rm in} = 45$ V under the (a) conventional control scheme and the (b) proposed TS fuzzy control scheme.

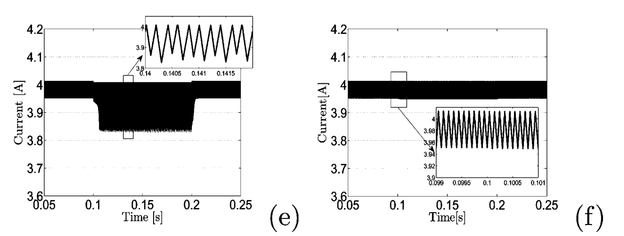

Fig. 15. Output current response of the converter subject to sudden load changing from 30 to 50 Ω and from 50 to 30 Ω at 0.1 and 0.2 s, respectively, with $V_{\rm in} = 45$ V and $I_{\rm ref} = 4$ A under the (a) conventional control scheme and the (b) proposed TS fuzzy control scheme.

switch sets can still be easily outstretched to guarantee the third stability condition in Theorem 2. To clarify this, assume that the fuzzy states (x, m_i) and (x, m_k) belong to the same region Ω_q, satisfying the first and second stability conditions of Theorem 2. Consequently, according to the third stability condition, the subvector field can switch from the discrete state m_i to m_k or *vice versa* at the continuous state x, while still guaranteeing stability. The discontinuous Lyapunov function (16) is changed at the subvector-field switching since it is measured by the same local Lyapunov function V_q (not necessarily quadratic) at both states. Therefore, having a common Lyapunov function for different discrete states is attractive from the robustness point of view whenever plausible. Now, assume that the fuzzy states (x, m_i) and (x, m_k) belong to different regions $(x, m_i) \in \Omega_q$ and $(x, m_k) \in \Omega_r$, measured by local Lyapunov functions V_q and V_r, respectively. It is deduced from the third stability condition that the discontinuous Lyapunov function (16) decrease is contingent on the switching of a discrete state to guarantee stability. Based on the same deduction, if $V_r(x) \leq V_q(x)$, it is possible to switch the discrete state from m_i to m_j and, hence, the subvector field from $\sum_{j\in\{1,2,\ldots\}} w^j(x, m_i)(A^j(m_i)x + B^j(m_i))$ to $\sum_{j\in\{1,2,\ldots\}} w^j(x, m_k)(A^j(m_k)x + B^j(m_k))$ at the continuous state x. The set of fuzzy continuous states where the discrete state is allowed to change and still fulfill the third stability condition in Theorem 2 due to the energy decrease of the given Lyapunov function (in the same regions as well as changing regions due to a change of subvector field) is considered as acceptable switch regions, and the notation $\bar{S}_{i,k}$ will be used to indicate specifically where m_i may be switched to m_k. Formally, the sets $\bar{S}_{i,k}$ are defined by

$$\bar{S}_{i,k} = \{(x, m_i) \in \Omega_q \text{ and } (x, m_k) \in \Omega_r | V_r(x) \leq V_q(x)\}. \tag{39}$$

Obviously, the acceptable switch region $\bar{S}_{i,k}$ should be much larger than the nominal switch set $S_{i,k}$ which means that $S_{i,k} \subseteq \bar{S}_{i,k}$. The region defined by the union of $\bar{S}_{i,k}$ and $\bar{S}_{k,i}$ covers at least all fuzzy continuous states where switching from discrete states m_i to m_k are admitted due to the existing Lyapunov function. It is worth noting that this region would be larger than the nominal specified ones, possibly as a result of conservatively formulating the stability conditions to be valid in larger regions than necessary by the replacement of regions by quadratic forms. When discrete-state transitions $m_i \rightarrow m_k$ are possible in the entire continuous state space $\Re^n$, then $\bar{S}_{i,k} \cup \bar{S}_{k,i} = \Re^n$.

In the case of exponential stability, the estimate of the exponential convergence rate stays intact in the acceptable switching region, since the third condition of stability in Theorem 2 does not affect the estimate.

VII. Conclusion

The TS fuzzy approximation technique has been extended to model the switching nature of the nonsmooth dynamical model of the dc–dc power electronic boost converter. The modeling method is particularly appropriate for the analysis of the fast-scale stability of the voltage and current waveforms, i.e., the presence and onset of nonlinear phenomena occurring at clock frequency. It should be noted that the proposed model has also the capability of representing a possible sliding-mode behavior by introducing new discrete states associated with the sliding mode. Therefore, it can be applied to a much wider range of nonsmooth dynamical systems such as mechanical systems with sliding behavior or possibly systems with sliding-mode control. More importantly, the proposed modeling approach can reduce the need for the specialized sophisticated software packages for the accurate numerical simulation of nonsmooth systems due to the stiffness of their equations.

A rigorous mathematical stability analysis based on discontinuous Lyapunov functions is developed to study the exponential stability of the system and a variety of circuit nonlinear behaviors such as period-doubling bifurcation and chaotic operation. Stability conditions are presented as LMIs to automate the search for Lyapunov functions using existing and well-established interior-point numerical methods. The resulting stability conditions provide an effective technique to predict the onset of fast-scale instabilities.

The main advantage of the model-based TS fuzzy approach is demonstrated by the design of a switching fuzzy controller inspired by fuzzy-gain scheduling concepts and based on the new theorem. The new fuzzy control scheme has been shown to improve the fast-scale stability by extending the stable period-one region of the system to a significantly larger operating domain while simultaneously boosting the slow-scale transient response of the circuit.

APPENDIX

Proof of Theorem 2: First, assume that $\alpha : \Re^+ \rightarrow \Re^+$ and $\beta : \Re^+ \rightarrow \Re^+$ are class $\mathcal{K}$ functions (see [7] for the definition of class $\mathcal{K}$ functions). To establish stability in the sense of Lyapunov, it must be shown that, for any $R > 0$ (any $R > 0$ such that $B_R(0)$ is included in Ω_x), there exists $r(R) > 0$ such that $\|x_0\| < r$ implies that $\|x(t)\| < R$ for all $t \geq 0$. Due to the first condition and the continuity of class κ functions, for any $R > 0$, there exist $r(R) > 0$ such that $\beta(r) > \alpha(R)$. Let the initial state $(x_0, m_0) \in \mathcal{F}_0$ be chosen such that $\|x_0\| < r$. Let t_k denote the consecutive times when the trajectory passes from one region to another. According to the second condition, the energy decreases in every region, and according to the third condition, the energy decreases at every switching time. Hence, $V(x(t)) \leq V(x_0)$ for all $t \geq 0$; this will be proved formally using the following exponential stability. Thus,

$$\alpha(\|x(t)\|) \leq V(x(t)) \leq V(x_0) \leq \beta(\|x_0\|) \leq \beta(r) < \alpha(R) \tag{40}$$

implying that $\|x(t)\| < R$ for all.

To prove the exponential stability, it must be shown that there exist two positive numbers, namely, $k_1 > 0$ and $k_2 > 0$, such that

$$\|x(t)\| \leq k_1 \mathrm{e}^{-k_2 t} \|x_0\|, \qquad t \geq 0. \tag{41}$$

Let k_1 and k_2 be defined as in (41). If it is shown that

$$V(x(t)) \leq V(x_0)\mathrm{e}^{-2k_2 t}, \qquad t \geq 0 \tag{42}$$

is true, then (41) follows from the first condition of the theorem. Hence, it remains to show that (42) holds.

Assume that the trajectory is in region Ω_p in the time interval $t \in [0 \quad t_1)$. Then, $V(x(t)) = V_p(x(t))$, $t \in [0 \quad t_1)$. Using the first and second conditions, we have

$$\dot{V}(x) \leq -\gamma \|x\|^2 \leq -\frac{\gamma}{\beta} V(x). \tag{43}$$

Consequently

$$V(x(t)) \leq V(x_0)\mathrm{e}^{-(\gamma/\beta)t} = V(x_0)\mathrm{e}^{-2k_2 t}, \qquad t \in [0 \quad t_1). \tag{44}$$

If t_1 is infinite, meaning that the trajectory never leaves the region, then (42) is true. Otherwise, assume that the trajectory passes through different regions and stays in Ω_q for $t \in (t_k \quad t_{k+1})$. Assume that $V(x(t)) \leq V(x_0)\mathrm{e}^{-2k_2 t}$, where $t \geq 0$ and $t \in (t_k \quad t_{k+1})$. Suppose that the trajectory reaches $\Lambda_{q,r}$ at time $t_{k+1} > t_k$ and stays in region Ω_r for $t \in (t_{k+1} \quad t_{k+2})$, where t_{k+2} may be infinite. Similarly, it can be shown that

$$V(x(t)) \leq V(x(t_{k+1}))\mathrm{e}^{-2k_2(t-t_{k+1})}, \qquad t \in (t_{k+1}\ t_{k+2}). \tag{45}$$

The third condition implies that

$$V_r(x(t_{k+1}+\varepsilon)) \leq V_q(x(t_{k+1}-\varepsilon)), \qquad \text{where } \varepsilon > 0;\ \varepsilon \rightarrow 0. \tag{46}$$

Therefore

$$\begin{aligned} V(x(t)) &\leq V(x(t_{k+1}))\mathrm{e}^{-2k_2(t-t_{k+1})} \\ &\leq V(x_0)\mathrm{e}^{-2k_2 t_{k+1}}\mathrm{e}^{-2k_2(t-t_{k+1})} \\ &= V(x_0)\mathrm{e}^{-2k_2 t}, \qquad t \in (t_{k+1} \quad t_{k+2}). \end{aligned}$$

Since $V(x(t)) \leq V(x_0)\mathrm{e}^{-2k_2 t}$ is true for $t \in [0 \quad t_1)$ and $V(x(t)) \leq V(x_0)\mathrm{e}^{-2k_2 t}$, $t \in (t_k \quad t_{k+1})$, implies that $V(x(t)) \leq V(x_0)\mathrm{e}^{-2k_2 t}$, $t \in (t_k \quad t_{k+2})$, it can be concluded from the principle just stated that (42) is true. If the assumptions hold globally, all inequalities hold for all initial states since the conditions imply that the trajectories remain in the bounded region defined by $V(x) \leq V(x_0)$. If $R_c = \{(x, m) \in \mathcal{F} | V(x) \leq c\}$ and $R_c \subseteq \Omega$, then every trajectory starting in R_c remains in R_c for all future times and satisfies (41). Hence, $R_c \subseteq R(k_1, k_2)$. □

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