Modelling and Analysis of Indirect Field-Oriented Control of SVPWM-Driven Induction Motor Drive Based on a Voltage Source Inverter

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Abstract—This paper presents the design and analysis of an indirect field-oriented controlled (FOC) induction motor drive system based on the space vector pulse width modulation (SVPWM) technique. The induction motor is fed to a voltage source inverter (VSI). The PI-controller has been employed for regulating the fluctuation of the motor current and torque, due to the parametric variation of an induction motor. As long as the FOC algorithm maintains the motor efficiency in a wide range of speeds, it also considers the torque changes with transient phases by processing a dynamic model of the motor. The Simulink-based model has been thoroughly developed in order to verify the accuracy and stability of the control system.

Index Terms—Induction motor, field oriented control, PI controller, Simulink, space vector modulation, vector control

I. INTRODUCTION

The induction motor (IM) is the workhorse and a leading candidate to be used in many industrial applications due to its ruggedness, reliability, robustness, lower price and higher efficiency in comparison with other motor types. Despite its wide range of advantages, employing the induction motor is challenging due to its complex mathematical model, the temperature dependent structure that leads to the electrical parameter oscillation and its nonlinear behaviour during saturation. Therefore, controlling the induction motor needs a high-performance control algorithm such as “vector control”.

A simpler form of IM control technique is so-called “scalar control” which using the non-vector controlled drive schemes. Although a steady-state operation of an induction motor can be guaranteed by a simple voltage fed, current controlled or speed controlled schemes, but its transient behaviour due to its slow response is not ideal. Moreover, despite the fact that the scalar control method such as constant volts per hertz (V/f) control is simple and low-cost, but its practical application at low frequency is still challenging, due to the influence of the dead time, the stator resistance and the necessary rotor slip to produce torque state response [1].

In vector control technique, not only the magnitude of the current and voltage are controlled, but also the phase of each current and voltage are controlled that results in a more accurate control system [2].

The principle of field orientation has introduced by F. Blaschke in 1972 [3] based on the decoupled control of the torque and flux in the motor. Briefly, the field-orientated control (FOC) consists of controlling the stator currents represented by a vector. The stator current components are converted into a rotating frame, which then rotates with flux linkage spaced vectors. One of the most effective vector control of induction motor is the indirect field oriented control (IFOC) due to its simplicity in designing and construction. In this technique, the rotor flux and torque which are generating current components of the stator current must be decoupled in order to obtain the high performance of the torque and speed in an IM drive. Furthermore, by incorporating the PI control system and using space vector modulation (SVM) technique results in the better performance of the decoupled control system in both transient and steady state conditions.

The IFOC uses a machine model that leads to a highly dependent behaviour of system on machine parameters such as the stator voltages and current, the initial angular frequency, the magnetising currents, the rotor fluxes and the rotor speed. Prior to implementing the IFOC of IM drives using SVM, these parameters are necessary to be measured in order to perform the control technique. Among all these parameters, the rotor fluxes and magnetising currents are difficult to measure. Therefore, the estimators are deployed in the control technique in order to overcome this issue. The accuracy and performance efficiency of the IFOC algorithm are mainly dependent on this stage and the deployed tuned PI controllers. In this work, the model-based IFOC system using SVM technique of a three-phase, four poles squirrel-cage IM is presented and analysed.
II. FIELD-ORIENTED CONTROL

A. Mathematical Model of an Induction Motor

The three-phase voltages, currents and fluxes of AC-motors can be analysed in terms of complex space vectors \[4\]. In order to investigate the large currents, produces voltage dips and oscillatory torques during the startup and other transient operations, the induction motor dynamic is modelled in the higher order mathematical equations to have a reliable model. In the modelling of an induction motor, the three-phase supply and current are converted to two-phase in the stationary reference frame known as direct axis (i.e., d-) and quadrature (i.e., q-) axis to simplify the analysis of three-phase circuit \[5\]. This d-q transformation has been carried out with the help of Parks transformation matrix \[6\].

![Fig. 1. The d-q equivalent circuit model of an induction motor.](image)

By referring to a generalised d-q equivalent circuit model of an induction motor drive as shown in Fig. 1, the d-q transformations applied to the three-phase stator voltages are as follows:

\[
\begin{align*}
v_{sd} &= R_s i_{sd} + \frac{d}{dt} \lambda_{sd} - \omega_c \lambda_{sq} , \\
v_{sq} &= R_s i_{sq} + \frac{d}{dt} \lambda_{sq} - \omega_c \lambda_{sd} , \\
v_{rd} &= R_r i_{rd} + \frac{d}{dt} \lambda_{rd} - \omega_s \lambda_{rq} , \\
v_{rq} &= R_r i_{rq} + \frac{d}{dt} \lambda_{rq} - \omega_s \lambda_{rd} ,
\end{align*}
\]

(1)

where \(\lambda_{sd}\) and \(\lambda_{sq}\) are the stator flux components of d- and q-axis respectively presented as follows:

\[
\begin{align*}
\lambda_{sd} &= L_s i_{sd} + L_m i_{rd} , \\
\lambda_{sq} &= L_s i_{sq} + L_m i_{rq} , \\
\lambda_{rd} &= L_r i_{rd} + L_m i_{ld} , \\
\lambda_{rq} &= L_r i_{rq} + L_m i_{qs} .
\end{align*}
\]

(2)

\(v_{sd}\) and \(v_{sq}\) are the d-; and q-axis stator voltage components, \(R_s\) is the stator resistance, \(i_{sd}\) and \(i_{sq}\) are the d-; and q-axis stator current components, respectively; \(L_s\) is the stator inductance, and \(\omega_c\) is the rotor electrical angular speed.

B. Field Oriented Control Technique

FOC converts the measured motor currents/voltages into the field current, and the torque one. There are two basic categories of FOC: the direct and the indirect method. The direct implementation is based on the direct measurement of the flux in stator-rotor gap, while indirect method estimates the magnetic flux from the applied voltages and resultant currents through the motor model. This leads to a highly dependent behaviour of the system on the machine parameters in contrast to direct methods. However, IFOC is the most common drive control for IM in the industry due to its considerable performance in comparison to the moderate amounts of parameter information that is used to be given to the controller. The key factor in FOC is the knowledge of the rotor flux position \(\theta_c\). Any error in this variable results in misaligning the rotor flux with the d-axis, and eventually causes \(i_{sd}\) and \(i_{sq}\) to be an incorrect flux and torque components of the stator current. The basic scheme of the torque control with IFOC is shown in Fig. 2. There are mainly two control loops that control induction motor drive, namely the internal pulse width modulation current control loop and external speed control loop \[7\]. The \(d^*, q^*\) axes are fixed on the stator while the \(d^*, q^*\) denoted as the fixed axes on the rotor, which are rotating with respect to the reference axes at synchronous speed \(\omega_s\).

The rotating axes with synchronous speed (i.e., \(d^*, q^*\)) are rotating ahead of the \(d^*, q^*\) axes by the positive slip angle (\(\theta_s\)) corresponding to the slip frequency (\(\omega_s\)) \[8\]. The \(\theta_c\) is calculated from \(\omega_s\) and the rotor speed (\(\omega_r\)) as shown in the Eq. 3.

\[
\theta_c = \int (\omega_c + \omega_s) dt = \theta_c + \theta_{sd} .
\]

(3)

where

\[
\omega_c = \frac{N_p}{2} \omega_m ,
\]

(4)

\[
\omega_{sd} = \frac{L_m i_{sq}}{\tau_r \lambda_r} .
\]

(5)

Eq. 4 defines the relation between the mechanical and electrical rotor speed (i.e., \(\omega_m\) and \(\omega_c\), respectively) considering this fact that the rotor pole (i.e., \(N_p\)) is directed on the \(d^*\) axis. In Eq. 5, \(\lambda_r\) is the estimated rotor flux linkage, \(L_m\) is the mutual inductance between the stator and the rotor, and \(\tau_r\) is denoted as the time constant of the rotor that is defined by \(\tau_r = \frac{T_r}{\omega_s}\). Based on the FOC concept, the stator current components (i.e., \(i_{sd}\) and \(i_{sq}\)) should be oriented in phase (flux component) and in quadrature (torque component) to the rotor flux vector (\(\lambda\)). By locking the phase of the reference system such that the rotor flux is always along the d-axis (flux axis), then

\[
\lambda_{rd} = \lambda_r \quad \text{&} \quad \lambda_{rq} = 0 .
\]

(6)

In the IFOC algorithm, q-axis stator current \(i_{sq}\) controls the torque and d-axis stator current \(i_{sd}\) controls the rotor flux independently. Thus, the d- and q-axis stator current reference \(i_{sd}^*\) and \(i_{sq}^*\) are calculated from the rotor flux reference input \(\lambda_r\) and command torque signal \(T_c\), respectively as follows.

\[
T_c = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \frac{L_m}{L_r} (\lambda_{rd} i_{sq} - \lambda_{rq} i_{sd} ) .
\]

(7)
As follows:

\[
i_{sd}^* = \frac{\lambda_r^*}{L_m},
\]

\[
i_{sq}^* = \frac{2}{3} \frac{L_r}{L_m} L_m T_e \lambda_r^*.
\]

where \(\lambda_r^*\) is the estimated rotor flux linkage that can be calculated by Eq. 10.

\[
\lambda_r^* = \frac{L_m i_{sd}}{1 + \tau_e s}.
\]

By reminding that IFOC is intended to make \(\lambda_{rq} = 0\) while the rotor direct flux \(\lambda_{rd}\) converges to the reference \(\lambda_{rd}^*\) [9] as well as considering the Eq. 4, torque equation can be simplified as follows:

\[
T_e = \frac{3}{2} \frac{P L_r}{L_m} (\lambda_{rd} i_{qs}) = \frac{3}{2} \frac{P L_m^2}{L_r} (i_{ds} i_{qs}).
\]

This equation resembles the concept behind the field oriented controlled IM drive. The decoupled torque and flux stator current components (i.e., \(i_{qs}\) and \(i_{ds}\)) are the main factors in controlling the motor torque. Thus the torque can be controlled by independently controlling these two currents.

### III. SPACE VECTOR PULSE WIDTH MODULATION

The space vector pulse width modulation (SVPWM) technique has become the most popular PWM technique for the control of AC motors fed by a three-phase voltage source inverters (VSI). This algorithm can decrease the total harmonic distortion (THD%) of the AC current/voltage waveform along with the higher bus voltage utilisation (i.e., 86%) [10]. Moreover, SVPWM results in less switching losses compared with standard PWMs, and has higher output quality while provides flexible control of the output voltage and output frequency for Variable Speed Drive (VSD). SVPWM approach also has the advantage of the ease of digital implementation [11]. However, implementing it in three-level converter is more complex because of the large number of inverter switching states.

Space vector modulation (SVM) for three-leg VSI is based on the representation of the three-phase voltage as vectors in a two-dimensional plane. Its operation principle is that while the upper switch in the converter leg is switched ON, the corresponding lower switch should be switched OFF, therefore eight switching states are available. SVM divides the vector plane into six sectors, each one is 60 degrees as shown in Fig. 3. The rotating reference vector can be approximated in each switching cycle by switching between the two adjacent active vectors and the two zero vectors. In order to implement the space vector modulation, a reference voltage \(V_z\) is sampled with a frequency \(f_s = \frac{1}{T_s}\) where \(T_s\) is the switching period. By using any two adjacent voltage vectors (i.e., \(V_1 - V_6\)) and one of the null vectors (\(V_0\) or \(V_7\)), the \(V_z\) is generated. The two vectors are time-weighted in a sample period \(T_s\) to produce the desired output voltage. The value of each vector, the associated time duration, and the angle between them (i.e., \(\theta_e\)) should be defined first to realise SVPWM as follows.

\[
V_z T_z = V_k T_k + V_{k+1} T_{k+1} + V_0 T_0.
\]

\[
T_k = \frac{V_z V_{k+1}}{V_{k+1} - V_k} \frac{2}{\sqrt{3}} \sin(60 - \theta).
\]

\[
T_{k+1} = \frac{V_z V_k}{V_{k+1} - V_k} \frac{2}{\sqrt{3}} \sin(\theta).
\]

\[
T_s = T_1 + T_2 + T_0.
\]

Where \(T_s\) is the switching period of SVM (sampling time), \(T_z\) is half the switching period and \(T_k\) is time duration for applying \(V_k\) in SVM, \(V_k\) is leg voltage of the inverter and \(k\) is the position of the sector in SVM.

### IV. SIMULINK IMPLEMENTATION OF THE IFOC IM DRIVE SYSTEM

In order to show the performances and the robustness of the presented control system, a detailed model for the IFOC IM driven with SVPWM technique has been carried out by numerical simulations using Matlab/Simulink. The machine parameters for modelling and implementing the control algorithm for the IM drive system that is fed by a three-phase VSI is listed in Table I. The VSI has been developed with the two-level six IGBT switches in parallel with diodes to allow the bidirectional current flow, as well as the unidirectional voltage...
TABLE I
THE INDUCTION MOTOR PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power 50 Hp</td>
<td>Rotor inductance 0.8e-3 H</td>
</tr>
<tr>
<td>Voltage 460 V</td>
<td>Mutual inductance 34.7e-3 H</td>
</tr>
<tr>
<td>Frequency 60 Hz</td>
<td>Moment of inertia 1.662 kg.m²</td>
</tr>
<tr>
<td>Stator resistance 0.087 Ω</td>
<td>Number of poles 4</td>
</tr>
<tr>
<td>Rotor resistance 0.228 Ω</td>
<td>Rotor type Squirrel cage</td>
</tr>
<tr>
<td>Stator inductance 0.8e-3 H</td>
<td>Friction Coefficient 0.01</td>
</tr>
</tbody>
</table>

blocking capability [12]. In order to summarise the operation behaviour of the controller system, a block diagram is given in the Fig. 4, and each step is explained in the following sections.

A. The Coordinate Transformation

The three-phase rotor current (i.e., \(i_a, i_b, i_c\)) has converted into the time-invariant d- and q-axis coordinate by using the Park and Clark transformation, and the inverse Park transformation. The implemented blocks in the Simulink are shown in Fig. 6 and Fig. 7.

B. The Rotor Flux Angle Calculation

The rotor flux position is defined and developed in Simulink as can bee seen in the Fig. 8. In order to calculate the rotor flux angle, the rotor flux value is needed itself. The estimated rotor flux (\(\lambda^*\)) is also modeled based on Eq. 10 and is shown in Fig. 9.

![Fig. 4. The IFOC of an IM drive system with SVPWM technique.](image)

![Fig. 5. The block diagram of the presented IFOC IM drive system in Simulink.](image)

![Fig. 6. The Simulink model of the Park and Clark transformation.](image)

![Fig. 7. The Simulink model of the inverse Park transformation.](image)

![Fig. 8. The rotor flux position calculation.](image)

![Fig. 9. The rotor flux estimation block.](image)
C. The Current and Speed Controller

The IFOC of an induction motor uses the induction motor model and the feedback of rotor speed to produce the decoupled phase current commands for the q-axis and d-axis components of stator current which eventually produces the torque and flux, respectively. The main objective of the stator current control system is the formation of the desired current values of the d- and q-axis. These currents after transferring to a three-phase voltage system, they define the triggering signals for the switches in the inverter legs, which are responsible for supplying the motor with an appropriate phase voltage. The output values of the stator current control unit are only the voltage references on d- and q-axis. The principle of forming the output voltage is based on using a PI-regulator, which is then fed the static error value between the reference and actual values for current in d- and q- axis. Upper level torque and the flux controllers (i.e., the speed and current controllers) define the reference values of the \( i_{d\text{ref}} \) and \( i_{q\text{ref}} \) currents. The developed Simulink model is shown in Fig. 10.

![Fig. 10. The current regulator.](image)

In the speed regulator block, the reference speed is set to 80 rad/s. This speed is compared with the actual rotor speed of the IM and deploying the PI controller with the gains and structure stated in [12], the reference torque component of the stator currents \( i_q \) is defined.

D. Space Vector Pulse Width Modulation (SVPWM)

The space vector modulation block defines the states of each inverter switches based on the location of the reference vector within the inverter output space. When the desired output voltage vector \( V \) is in one of the sector vector, \( V \) could be synthesised by the pulse-width modulation (PWM) of the two adjacent non zero switching state vectors, \( V_1 \) and \( V_2 \) with the duty cycle of each \( T_1 \) and \( T_2 \), respectively as well as using a zero vector \( (V_0 \text{ or } V_7) \) with \( T_0 \).

\[
T_1V_1 + T_2V_2 + T_0V_0 = V = mV_0 e^{j\alpha}.
\]  

(15)

where, \( 0 \leq m \leq 0.850 \) is the modulation index. This would correspond to a maximum line-to-line voltage of 1.0 Vg, which is 15% more than the conventional sinusoidal PWM [11]. The \( V_\alpha \) and \( V_\beta \) are inputs to the SVPWM block. The angle that can be used to select the particular sector in SVM is calculated as bellow.

\[
\theta = \tan^{-1} \frac{V_\beta}{V_\alpha}.
\]  

(16)

By implementing the Eq. 16, the phase leg voltage of the inverter is selected based on the sector obtained in Table II. The last step after defining the time duration for the applied leg voltage vectors and each sector as well as using the Eq. 17 ,the space vectors are derived by comparing the resulted signal with the triangular carrier signal.

\[
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} = \begin{bmatrix}
V_K \\
V_{K+1} \\
0
\end{bmatrix} \begin{bmatrix}
T_K \\
T_{K+1} \\
T_0 \end{bmatrix}. 
\]  

(17)

In this work, for employing the vector control of the induction motor in the simulation, the reference torque considered to be 200 Nm, the reference \( i_d \) is set to 40 A and the reference speed is 80 rad/s. The torque is imposed on the system after one second of running the simulation. In order to investigate the overall performance of the field oriented controller designed for the induction motor with the space vector modulation model, the specified parameters of the motor and SVM specifications of 20 kHz were used and the system has been simulated. The results are discussed in the next section.

V. Simulation Results and Discussions

The simulation model for indirect vector control of the induction motor with the PI speed controller was run for two seconds. The motor started from standstill at \( t = 0 \) and reached its rated speed of 80 rad/sec. Fig. 11 shows the simulated stator currents, while the simulated d- and q-axis stator current components are depicted in Fig. 12 and Fig. 13. Fig. 14 illustrates the electrical rotor speed with 80 rad/s as the reference speed; whereas the rotor electromagnetic torque is given in Fig.15. The stator current is three-phase with sinusoidal waveform and the rotor speed reaches the desired value in a smooth manner and therefore, it remains steady. The torque also sets at its reference value. The \( i_d \) and \( i_q \) currents track their reference values as well as having the DC values. This justified the concept of the FOC that the controllers have been designed into the d-q synchronously reference frame. To improve the smoothness of the current and torque responses, the sampling time could be increased. It is concluded qualitatively from the simulation results that the model performed according to the expectations.
VI. CONCLUSION

The primary emphasis of this work has been focused on the modelling, development, and analysis of the indirect field-oriented controlled IM drive system, as well as the step by step defining its Simulink-based implementation. The presented IFOC model would effectively result in the improvement of the implementation of sensor-less FOC induction motor drives. This model could also be potentially utilised to augment advanced controllers in terms of less total harmonic distortion, less switching losses and the simple practical implementation.

REFERENCES