Modeling and Stability Analysis of Closed Loop Current-Mode Controlled Ćuk Converter using Takagi-Sugeno Fuzzy Approach

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Abstract: The current-mode controlled Ćuk converter, being a fourth-order nonlinear non-smooth system, does not lend itself to any simple analysis. The difficulty lies mainly in the complex modeling of the circuit needed to capture all the essential nonlinearities that occur in fast time scale such as border-collision bifurcation and chaos. This paper extends the Takagi-Sugeno fuzzy modeling approach to capture these fast time-scale nonlinearities. Non-smooth Lyapunov theory is employed to study the stability of the system and to locate the operating point at which the converter loses its stable period-1 operation.

1. INTRODUCTION

DC-DC SWITCHING converters, with their inherent non-smooth characteristics, are a traditional benchmark for testing different nonlinear modeling and stability analysis methods. Closed loop current-mode controlled Ćuk converter are fourth-order nonsmooth dynamical systems that show a variety of nonlinear phenomena. To date, a number of of modeling methods have been proposed to capture all the nonlinearities in power electronic converters. These are primarily based on nonlinear discrete modeling of these systems using variety of mapping techniques such as the Poincaré and Henan maps Banerjee and Verghese (2001). Although powerful, these methods can be off limited practical use when applied to more complicated converter topologies like the Ćuk converter where accurate mathematical modeling is essential to identify all fast-time scale nonlinear phenomena neglected by traditional averaged modeling techniques which are only capable of characterizing the low-frequency behavior of the circuit Tse (2004). In exploring new methods for nonlinear modeling of the Ćuk converter, a novel Takagi-Sugeno fuzzy modeling approach is proposed in this paper. The proposed TS fuzzy modeling approach is shown to be able to accurately simulate the high-frequency behavior of the circuit without the need for extensive resources Mehran et al. (2008). Furthermore, the proposed TS fuzzy approach does not require dedicated software platforms and can be performed using widely available standard software tools, unlike other approaches based on discrete nonlinear modeling Tse (2004); Daho et al. (2008).

The traditional approach for the stability analysis of power electronic converters is to obtain the period-n equilibrium orbit or limit cycle by setting $x(0) = x(nT)$ acquired from the iterated map. The stability of the limit cycle can then be determined by checking if the magnitude of all characteristic multipliers at the limit cycle are less than unity Tse and Chan (1995); Tse (2004). However, using this classical approach when analyzing the behavior of the Ćuk converter would result in a fourth-order iterative map increasing the complexity of the stability analysis substantially . In an attempt to overcome this difficulty, the Filippov's method has been applied to study the stability of the free-running current controlled Ćuk converter Daho et al. (2008) with some success. This, however, does not entirely remove the difficulties associated with the complexity of the high-ordered maps employed in the analysis. As an alternative, the Takagi-Sugeno fuzzy model-based system is employed in this paper to investigate the stability of the nonsmooth model of the converter and predict the operating point at which the converter loses it's stability to a border collision bifurcation. The stability analysis is based on nonsmooth Lyapunov theory, formulated numerically as a Linear Matrix inequality (LMI) problem, appropriate for the nonsmooth nature of the model Shevits and Paden (1994). The new analysis has the advantage of less computational complexity and a more direct approach to the problem by checking the feasibility of the LMI system. In addition, the LMI stability conditions, which will be presented in this paper, can be used as the basis for a new control strategy to extend the period-1 behavior to a much wider range of operating circuit conditions. The method has been applied to non-autonomous systems like the dc-dc buck converter with promising results Mehran et al. (2008).

2. THE ĆUK CONVERTER AND ITS MATHEMATICAL MODEL

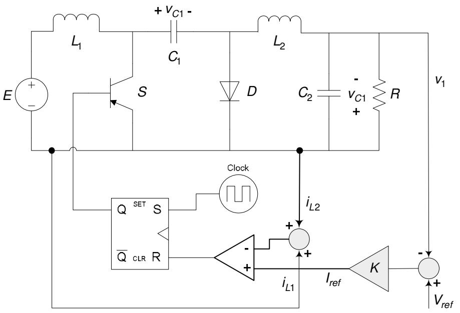


Fig. 1. Ćuk converter under original current-mode control scheme

The closed loop current-mode controlled Ćuk converter (Figure 1) is a non-autonomous, nonsmooth dynamical system that conventionally is controlled by comparing the sum of the inductor current, $i_{L1} + i_{L2}$, with a reference current $I_{\rm ref}$ to generate the OFF driving signal for the switch. The switch S is turned on at the beginning of the cycle $t = nT$ and stays on until $i_{L1} + i_{L2}$ reaches the value of $I_{\rm ref}$ when it is turned OFF until the next cycle begins. The control scheme can be formulated as follows:

$$I_{ref} - (i_{L1} + i_{L2})_n = [\frac{E}{L_1} + \frac{v_{C2,n} - v_{C1,n}}{L_2}]d_n T \qquad (1)$$

where subscript n denotes values at $t = nT$ and

$$d_n = \frac{I_{ref} - (i_{L1} + i_{L2})_n}{(\frac{E}{L_1} + \frac{v_{C2,n} - v_{C1,n}}{L_2})T} \qquad (2)$$

The dynamics of the system can be described by four sets of differential equations:

$$\frac{dv_{C1}(t)}{dt} = \begin{cases} \dfrac{-1}{RC2}v_{C1} + \dfrac{1}{C2}i_{L1}, \text{ S is OFF} \\ \dfrac{-1}{RC2}v_{C1} + \dfrac{1}{C2}i_{L1}, \text{ S is ON} \end{cases} \qquad (3)$$

$$\frac{dv_{C2}(t)}{dt} = \begin{cases} \dfrac{-1}{C1}i_{L1}, \text{ S is OFF} \\ \dfrac{-1}{C2}i_{L2}, \text{ S is ON} \end{cases} \qquad (4)$$

$$\frac{di_{L1}(t)}{dt} = \begin{cases} \dfrac{-1}{L2}v_{C1} + \dfrac{1}{L2}v_{C2}, \text{ S is OFF} \\ \dfrac{-1}{L2}v_{C1}, \qquad \text{S is ON} \end{cases} \qquad (5)$$

$$\frac{di_{L2}(t)}{dt} = \begin{cases} \dfrac{1}{L1}v_{in}, \qquad \text{S is OFF} \\ \dfrac{-1}{L1}v_{C2} + \dfrac{1}{L1}v_{in}, \text{ S is ON} \end{cases} \qquad (6)$$

If we define the state vector $x = [v_{C1}\ v_{C2}\ i_{L1}\ i_{L2}]$ and u as an input voltage V_{in}, equations (3), (4), (5) and (6) can be written as:

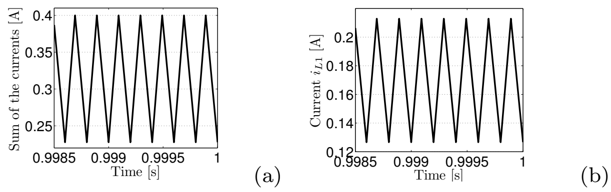


Fig. 2. Nominal period-1 operation of the Ćuk converter with $I_{\rm ref} = 0.4A$ for (a) $i_{L1} + i_{L2}$ and (b) i_{L1} (fixed parameter values are $V_{\rm in} = 15V$, $L1 = L2 = 1mH$, $R = 75\Omega$, $C = 47\mu F$, $T = 1/2500s$).

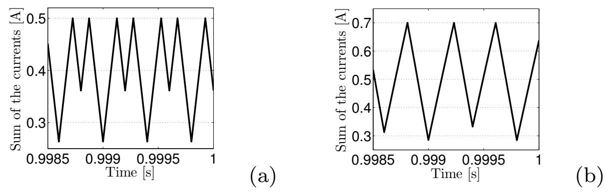


Fig. 3. Nonlinear operation of the Ćuk converter (a) period-2 operation with $I_{\rm ref} = 0.5A$. (b) chaotic operation with $I_{\rm ref} = 0.7A$ (fixed parameter values are $V_{\rm in} = 15V$, $L1 = L2 = 1mH$, $R = 75\Omega$, $C = 47\mu F$, $T = 1/2500s$).

$$\dot{x} = \begin{cases} A_1 x + Bu, \ (I_{\rm ref} - A(i_{L1} + i_{L2})) < i_{\rm ramp}(t), \\ A_2 x + Bu, \ (I_{\rm ref} - A(i_{L1} + i_{L2})) > i_{\rm ramp}(t). \end{cases}$$

where:

$$A_1 = \begin{bmatrix} -1/RC2 & 0 & 1/C2 & 0 \\ 0 & 0 & -1/C1 & 0 \\ -1/L2 & 1/L2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L1 \end{bmatrix} \qquad (7)$$

$$A_2 = \begin{bmatrix} -1/RC2 & 0 & 1/C2 & 0 \\ 0 & 0 & 0 & 1/C1 \\ -1/L2 & 0 & 0 & 0 \\ 0 & -1/L1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L1 \end{bmatrix} \qquad (8)$$

In this paper, the proportional feedback controller is used in which $i_{\rm ref} = K(V_{\rm ref} - v_1)$ where K is the control parameter and $V_{\rm ref}$ is the reference voltage. The outer closed loop determines the reference current $I_{\rm ref}$ based on the value of v_1. Under this control scheme, the normal output of the converter will be a periodic current waveform with a peak value of $I_{\rm ref}$ and a period that is equal to the period of the PWM ramp signal as shown in Figures 2a and 2b. Figure 3a and Figure 3b show converter unstable period-2 operation resulting from a border collision bifurcation and chaotic behavior, respectively, as the value of reference current is varied.

The example Ćuk converter has two switching manifolds corresponding to the ON and OFF switching instances. Figure 4 depicts the stable period-1 orbit in $v - i$ space, where the orbit periodically switches between X(dT) and the fixed point of the cycle with Poincaré map X(0). If a system parameter like $I_{\rm ref}$ is varied, the circuit becomes unstable through a period-doubling bifurcation and then chaos, as demonstrated in Figure 5 Tse (2004).

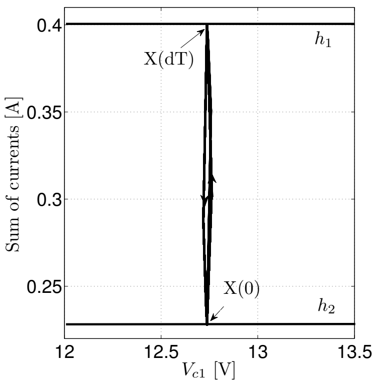


Fig. 4. Period 1 limit cycle:Phase space and the transversal intersection

3. TS FUZZY MODEL OF THE BUCK CONVERTER FOR FAST-SCALE ANALYSIS

The fuzzy inference system of Takagi-Sugeno Fuzzy models Takagi and Sugeno (1985); Tanaka (2001) used to approximate *smooth* dynamical functions is generally described by a set of rules in the form

$$\text{Rule } j : \text{IF } x_1 \text{ is } F_1^j \text{ AND...AND } x_q \text{ is } F_q^j$$
$$\text{THEN } \dot{x} = A^j x + B^j u, \quad j = 1, \ldots, l$$

and the dynamics of this system can be described by:

$$\dot{x} = \sum_{j=1}^{l} w^j(x)(A^j x + B^j u) \tag{9}$$

where $w^j(x)$ are normalized membership functions of the rule antecedents satisfying $0 \leq w^j(x) \leq 1$, $\sum_{j=1}^{l} w^j(\theta) = 1$ and l is the number of rules. It is well-known that the model structure above is the universal approximator of smooth nonlinear functions to arbitrary accuracy Tanaka (2001); Bergsten (2001). Approximating a nonsmooth dynamical function such as the Ćuk converter with a TS fuzzy model structure (9) cannot be realized Mehran et al. (2008). The TS model (9) designed to approximate smooth nonlinear functions is, fundamentally, incapable of representing the inherent discontinuity already observed in equations (3),(4),(5) and (6).

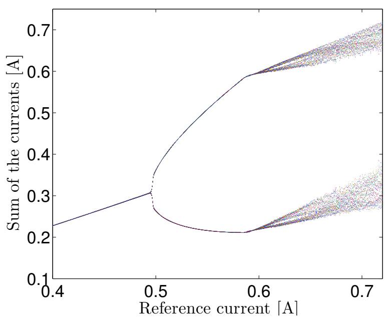


Fig. 5. Bifurcation diagram of the Ćuk converter under closed loop current-mode control (Figure 1), varying reference current as a parameter.

From the mathematical point of view, the existence (and uniqueness) of the TS fuzzy model of the form (9) would not be held, should the Ćuk converter's non-smooth dynamics be modeled with this approach Mehran et al. (2008). Approximating the converter with the smooth TS fuzzy model (9) by deriving the averaged model of the system would simply fail to observe all the fast-scale nonlinearities that occur at clock frequency Tse (2004). In order to be able to describe these switching events and capture the afore-mentioned nonsmooth nonlinear behaviors, the TS fuzzy modeling approach (11) has been extended to represent the essential dynamics (discontinuous) of the Ćuk converter Mehran et al. (2008), which has the form:

$$\text{Rule } j \;:\; \text{IF } (x_3 + x_4) \text{ is } F^j \tag{10}$$
$$\text{THEN } \dot{x} = \begin{cases} A^j(m_i)x + B^j(m_i)u \\ m^+ = \phi(x, m), \quad j = 1, 2, \quad i = 1, 2 \end{cases}$$

and by appropriately restricting the inference parameters, the dynamics of the discontinuous fuzzy system can be described by:

$$\begin{cases} \dot{x} = \displaystyle\sum_{j=1}^{l} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u) \\ m^+ = \phi(x, m) \end{cases} \tag{11}$$

where the function $\phi : \Re^n \times M \rightarrow M$ describes the dynamics of the discrete state associated with each nonsmooth switching. The notation m^+ means the next state of m. $x \in R^n$ is the continuous state, $m \in M = \{m_1, m_2\}$ is the discrete state, $A^j(m_i) \in \Re^{n\times n}$, $B^j(m_i) \in \Re^n$, $w^j : \Re^n \times M \rightarrow [0\ 1]$, $j \in I_l$, are continuous weighting functions which satisfy $\sum_{j=1}^{l} w^j(x, m) = 1$, l is the number of fuzzy rules and F^j are fuzzy sets. The state space is the Cartesian product $\Re^n \times M$. Any value of discrete state $m_i \in M$ is associated with an affine subsystem like:

$$\text{if } \forall x \in A(m_i)x + B(m_i)u \text{ then } m_i \in M,\ i \in \{1, 2\}$$

Remark 1: In general a value of m_i could be associated with a subset of subsystem as:

if $\forall x \in \{\sum_{j\in\{1,2,\ldots\}} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u)\}$ then $m_i \in \{m_1, m_2, \ldots, m_N\}$ when N is possibly infinite □

As inferred from (10) and (11), function ϕ formally describes the transition of the discrete states. Each transition involves jumping from the subset of fuzzy subsystems to another subset of fuzzy subsystems (from now on we shall refer to this as a fuzzy sub-vector field). Alternatively, the transition can be defined by a set of *switch sets* used to define the switching manifold mathematically. The general formulation of the *switch sets* is defined as:

$$S_{i,k} = \{x \in R^n | m_k = \phi(x, m_i)\},\ m_i \neq m_k,\ i, k \in I_N \tag{12}$$

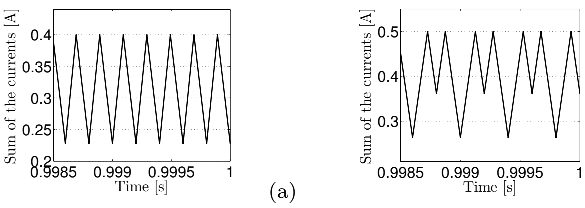
The switching hypersurface in the case of the Ćuk converter corresponds to the instant at which the switch S turned OFF

$$h(X, (dT)) = x_3(dT) + x_4(dT) - K(V_{\text{ref}} - x_1) \tag{13}$$

this can be described in the form of switch sets as:

$$\begin{aligned} S_{1,2} &= \{x \in R^n | x_3(dT) + x_4(dT) + Kx_1 < KV_{\text{ref}}\}, \\ S_{2,1} &= \{x \in R^n | x_3(dT) + x_4(dT) + Kx_1 > KV_{\text{ref}}\}\} \end{aligned} \tag{14}$$

where d is the duty ratio at each instant and T is the switching period.



(a)

(b)

Fig. 6. (a) The nominal period-1 operation of the TS fuzzy model of the Ćuk converter under original current-mode control when $I_{\mathrm{ref}} = 0.4A$ (b) The nominal period-2 operation of the TS fuzzy model of the converter when $I_{\mathrm{ref}} = 0.5A$.

To construct the TS model (10) of the converter, the membership functions are defined to exactly represent each fuzzy sub-vector field as follows:

$$F^1(x_3(t)+x_4(t)) = \frac{1}{2}(1+\frac{X_1(0)-x_3(t)-x_4(t)}{2l}),$$
$$F^2(x_3(t)+x_4(t)) = \frac{1}{2}(1+\frac{X_1(0)-x_3(t)-x_4(t)}{2l})$$

Here, l is a constant denoting the range of $i_{L1}(t)+i_{L2}(t) \in \{0.4-l, 0.4+l\}$ (in this analysis $l = 0.2$). The state vector $X(0) = [12.7187\ 27.5608\ 0.1012\ 0.1268]$ is the transversal intersection of the periodic orbit with the switching manifold when the switch is OFF (see Figure 4). In constructing the membership functions, we would normally opt for the equilibrium point of the system if using the smooth or averaged model of the converter Lian et al. (2006). However, our target in this analysis is a nonsmooth TS fuzzy model so the transversal intersection point should be taken into consideration when constructing the membership function to accurately represent switchings between two sub-vector fields. Since the vector fields of the system as expressed (7) and (8) are affine, they can be readily used in building the exact TS fuzzy models of the converter as $A^1(m_1) = A^2(m_1) = A_1$, $A^1(m_2) = A^2(m_2) = A_2$ and $B(m_1) = B(m_2) = B$ where the discrete states m_1 and m_2 actually represent the ON and OFF states of the converter, respectively. Compared with the original behavior of the converter(Figure 2 and 3), the proposed TS fuzzy model under current-mode control can recreate the nonsmooth switchings and capture all nonlinear phenomena, as shown in Figures 6a, 6b and 7.

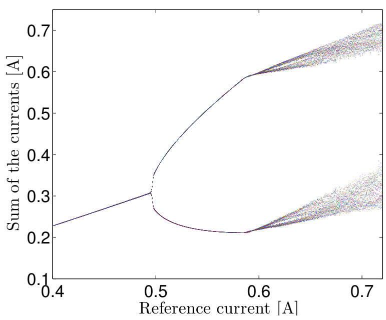


Fig. 7. Bifurcation diagram of the Ćuk converter modeled with TS fuzzy approach, varying reference current as a parameter.

4. EXPONENTIAL STABILITY ANALYSIS

The stability of the limit cycle in power electronics circuits is predominately determined using Poincaré maps Banerjee and Verghese (2001). At a selected switching surface, a perturbation of the switching point is mapped through the switching cycle back to the same switching surface. This is a discrete mapping, where the eigenvalues of the mapping matrix determines the stability of the limit cycle. Employing this approach with an engagement of Fillipov's method , the stability analysis of an autonomous Ćuk converter has already been studied Daho et al. (2008).

4.1 Nonsmooth Lyapunov method to show the stability of limit cycle

Considering the fact that the proposed TS fuzzy model of the converter represents a nonsmooth dynamical system, we introduce a novel Lyapunov method in this section, for the bifurcation analysis of the example converter. The traditional approach of finding a smooth global Lyapunov function to show the stability of TS fuzzy models approximating a smooth nonlinear system have been around for a long time in the model-based fuzzy control literature Tanaka (2001). Finding global quadratic Lyapunov candidates in the entire fuzzy state space, is problematic for the subsequent conservative LMI formulation even for the TS fuzzy models approximating smooth nonlinear functions Johansson et al. (1999). So naturally, for the proposed TS fuzzy model (11), this has proved unrealizable. Mehran et al. (2008). Briefly, to relax the Linear Matrix Inequalities (LMI) formulation for the stability analysis of the nonsmooth model of the converter, the Lyapunov function candidates should be given as discontinuous or nonsmooth functions. We also permit the fuzzy state space to be partitioned to different *flexible* regions. A switching function partitions the state space into two regions, separated by the switching surface $h(X,(dT)) = 0$. Following on, we let the fuzzy state space $\mathcal{F}$ be partitioned into two detached regions $\Omega_q, q \in I_2$ where $I_2 = \{1,2\}$. Therefore

$$\begin{array}{l}\Omega_1 = \{(x,m) \in \mathcal{F}|\ x \in \Re^n, m = m_1\} \\ \Omega_2 = \{(x,m) \in \mathcal{F}|\ x \in \Re^n, m = m_2\}\end{array} \tag{15}$$

A trajectory initiated in any region at time $t_k, k = 1,2,...$ can pass through another region if $t_k < t_{k+1}$. We define Λ_{qr} as a neighboring region which means:

$$\Lambda_{qr} = \{x \in \Re^n | \exists t < t_0, \text{ such that } x(t^-) \in \Omega_q, x(t) \in \Omega_r\} \tag{16}$$

Λ_{qr} is given by the hypersurface of the form (13). In fact, conditions for the switching from mode m_1 to mode m_2 at time τ are

$$\begin{cases} h(X,\tau) = 0 \\ \displaystyle\sum_{j\in\{1,2,...\}} w^j(x,m_i)(A^j(m_i)x + B^j(m_i)u)\cdot\nabla h(X,\tau) < 0 \end{cases} \tag{17}$$

Hence, if $\Lambda_{qr} \neq \emptyset$, Ω_q and Ω_r must be neighboring sets. As a sufficient condition, allow:

$$\mathrm{I}_\Lambda = \{(q,r)|\Lambda_{qr} \neq \emptyset\} \tag{18}$$

which is a set of tuples indicating that there is at least one point for which the trajectory passes from Ω_q to Ω_r.

Now, we define a local quadratic Lyapunov function for each region, which has the structure:

$$V(x) = V_q(x) = \tilde{x}^T \tilde{P}_q \tilde{x} \text{ when } (x, m) \in \Omega_q \tag{19}$$

where $\tilde{x} = \begin{bmatrix} x \\ 1 \end{bmatrix}$, $\tilde{P}_q = \begin{bmatrix} P_q & p_q \\ p_q^T & \pi_q \end{bmatrix}$, $\pi_q \in \Re$, $p_q \in \Re^n$, $P_q = P_q^T \in \Re^n \times \Re^n$ and $q \in I_2$.

Let Ω_q^x denote the continuous state of x in Ω_q. $V_q : \mathbf{cl}\Omega_q^x \rightarrow \Re, q \in I_\Delta$, is a (scalar) function which is assumed to be continuously differentiable on closure of region Ω_q (cl. denotes the closure of a set, which is the smallest closed set containing the set). In fact, the scalar function $V_q(x,t)$ is used to measure the fuzzy system's energy in a local region Ω_q. As implied, the overall Lyapunov function V (19) is a discontinuous Lyapunov function at the hypersurface (13) or at the neighboring regions Λ_{qr}, $(q,r) \in I_\Lambda$. Assuming $t_k < t_{k+1}$ for every trajectory with initial point in any region, $V(x)$ is *piecewise continuous* function with respect to time.

4.2 LMI formulation for stability and bifurcation analysis

The first stage in formulating the Lyapunov stability conditions into LMI conditions is to define fuzzy state-space partitions by (positive) quadratic functions. This is realizable using the so-called S-procedure technique to substitute the confined conditions with unconfined conditions Boyd et al. (1994). To explain the procedure in general terms, let $Q_0, \ldots, Q_s$, be quadratic functions of the variable $x \in \Re^n$ of the form:

$$Q_k(x) = x^T Z_k x + 2c_k^T + d_k,\ k = 0, \ldots, s, \tag{20}$$

where $Z_k = Z_k^T$. We consider the following condition on Q_0:

$$Q_0(x) \geq 0 \text{ in the region } \{x \in \Re^n | F_k(x) \geq 0, k \in I_s\} \tag{21}$$

The confined condition (21) can be substituted by an unconfined condition in the following way:

Lemma Boyd et al. (1994): if there exist $\delta_k \geq 0$, $k \in I_s$, such that

$$\forall x \in \Re^n, Q_0(x) \geq \sum\nolimits_{k=1}^{s} \delta_k Q_k(x) \tag{22}$$

then (21) holds. Hence, by introducing additional variables $\delta_k \geq 0$, $k \in I_s$, condition (21) can be turned into an LMI which can be written as:

$$x^T \begin{bmatrix} Z_0 & c_0 \\ c_0^T & d_0 \end{bmatrix} x \geq \sum\nolimits_{k=1}^{s} \delta_k x^T \begin{bmatrix} Z_0 & c_0 \\ c_0^T & d_0 \end{bmatrix} x \tag{23}$$

The replacement of (21) by **Lemma** may be conservative. However it can be shown that the converse is true in case of single quadratic form, *s=1* Boyd et al. (1994) contingent on the existence of some x such that $Q_1(x) > 0$. In case of hypersurface which can be defined by $Q_k(x) = 0$, $k \in I_s$, Lemma is true without the restriction $\delta_k \geq 0$.

Here, all the conditions in the stability theorem are immediately described by $Q_0(x) \geq 0$, where $Q_0(x)$ is a quadratic function defined by (20).

$$\begin{cases} Q_0(x) = \tilde{x}^T(\tilde{P}_q - \alpha\tilde{I})\tilde{x} \geq 0,\ \forall x \in \Omega_q^x,\ q \in I_2 \\ Q_0(x) = \tilde{x}^T(\beta\tilde{I} - \tilde{P}_q)\tilde{x} \geq 0,\ \forall x \in \Omega_q^x,\ q \in I_2 \\ Q_0(x) = -\tilde{x}^T(\tilde{A}(m)^T\tilde{P}_q + \tilde{P}_q\tilde{A}(m) + \gamma\tilde{I})\tilde{x} \geq 0, \\ \qquad \forall (x,m) \in \Omega_q \\ Q_0(x) = \tilde{x}^T(\tilde{P}_r - \tilde{P}_q)\tilde{x} \geq 0,\ \forall x \in \Lambda_{qr}^x,\ (q,r) \in \mathrm{I}_\Lambda \end{cases} \tag{24}$$

In the conditions above, α and β are constants which originally represent class $\mathcal{K}$ function $\alpha(\|x\|)$, $\alpha : \Re^+ \rightarrow \Re^+$ and $\beta(\|x\|)$, $\beta : \Re^+ \rightarrow \Re^+$(for definition of class $\mathcal{K}$ function see Khalil (1996)) and $\gamma > 0$ is a scalar constant. The third condition is satisfied on the hypersurface Λ_{qr}^x which can be given by $Q_k(x) = 0$, $k \in I_s$, where each $Q_k(x) = 0$ has the form (20) with no limitation on δ_k as mentioned before. All the conditions can be substituted by the unconfined condition (23); however, if the switching manifold cannot be represented exactly by $Q_k(x) = 0$, $k \in I_s$ in the last condition, it is possible to represent such a region with quadratic functions satisfying $Q_k(x) \geq 0$, in which the additional variables δ_k should be limited to $\delta_k \geq 0$.

Now, all the stability conditions for bifurcation analysis can be recast to LMI conditions:

LMI problem: If there exist $\tilde{P}_q$, $q \in I_\Delta$, constants $\alpha > 0$, $\mu_k^q \geq 0$, $\nu_k^{qij} \geq 0$, η_k^{qr} and a solution to *min* β subject to the three conditions:

- $\alpha\tilde{I} + \sum_{k=1}^{s_q} \mu_k^q \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix} \leq \tilde{P}_q$

  $\tilde{P}_q \leq \beta\tilde{I} + \sum_{k=1}^{s_q} \mu_k^q \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix}$, $q \in I_\Delta$
- $(q,i,j) \in I_\Omega$, $(\tilde{A}^j)^T\tilde{P}_q + \tilde{P}_q\tilde{A}^j$

  $+\sum_{k=1}^{s_{qij}} \nu_k^{qij} \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix} \leq -\tilde{I}$, $q \in I_\Delta$
- $\tilde{P}_r \leq \tilde{P}_q - \sum_{k=1}^{s_{qr}} \eta_k^{qr} \begin{bmatrix} Z_k^{qr} & c_k^{qr} \\ (c_k^{qr})^T & d_k^{qr} \end{bmatrix}$, $(q,r) \in I_\Lambda$

Then the fixed point [1] is exponentially stable in the sense of Lyapunov (see Mehran et al. (2009) for the proof).

For fixed parameter values as stated in Figure 2, the converter operates in a stable period-1 mode for a reference current value of $I_{\text{ref}} = 0.42$ as apparent from the bifurcation diagram in Figure 7. The system converges exponentially to the stable limit cycle, which will be verified by solving the LMI problem.

$$\tilde{\mathbf{P}}_\mathbf{1} = \begin{bmatrix} 43.8488 & 41.3433 & -93.4600 & 0 & 0 \\ 41.3433 & 42.5052 & -90.0070 & 0 & 0 \\ -93.46 & -90.00 & 810.9474 & 0 & 0 \\ 0 & 0 & 0 & 1.33 & -833.36 \\ 0 & 0 & 0 & -833.36 & 2.08 \end{bmatrix} \tag{25}$$

$$\tilde{\mathbf{P}}_\mathbf{2} = \begin{bmatrix} 1.0210 & 0 & -1.8009 & 0 & 0 \\ 0 & 1.6300 & 0 & -0.0050 & -24.44 \\ -1.8009 & 0 & 269.87 & 0 & 0 \\ 0 & -0.0050 & 0 & 554.86 & -0.14 \\ 0 & -24.44 & 0 & -0.147 & 949.35 \end{bmatrix} \tag{26}$$

with the optimal value of $\beta = 2.43$. If the value of the reference current is changed to $I_{\text{ref}} = 0.5$, the LMI problem is found infeasible and no optimal value of β can be

[1] For the Ćuk converter, the fixed point is the intersection point of limit cycle with a stroboscopic map.

obtained. The infeasibility of the LMI problem, can detect the edge of the bifurcation. The LMI problem can find a feasible solution with the optimum value of β for all values of $I_{\text{ref}} < 0.5$. This readily determines the range of stable period-1 operation of the converter without using nonlinear discrete mapping methods.

Allowing the fuzzy state space to be partitioned into flexible detached region is of prime importance in determining the actual stability of the nonsmooth system. An attempt to let the overall Lyapunov function V (19) measure the system's energy for the entire fuzzy state space means a common Lyapunov function for all discrete states has to be sought. The neighboring region Λ_{qr} is then empty and and the system will be stable regardless of any switching. However, with a single partition, searching for the feasible solution to the LMI problem is not possible even for the stable period-1 operating range of the converter and the LMI problem would be misleading (Table 1). Partitioning the fuzzy state space into more regions may result in a slightly different optimal value of β but it can accurately show the stable region of the converter and the operating point when the border collision bifurcation occurs. Deciding a suitable region partitioning depends on the user. Nevertheless, numerous and complex partitions should be avoided in practice as verifying the stability conditions in the LMI problem may turn out to be infeasible because of the resulting high computational complexity.

Table 1. Number of partitions and the corresponding LMI feasibility.

Number of Partitions in $\mathcal{F}$	LMI feasibility	Optimum value of β	Numerical complexity
1	not feasible	N/A	low
2	feasible	2.4330	low
4	feasible	2.4340	medium
8	feasible	2.4540	medium
16	feasible	2.4650	high
32	not feasible	N/A	very high

5. CONCLUSION

The Takagi-Sugeno fuzzy modeling approach has been specifically synthesized to incorporate the typical switching events of power electronic converters. Using the example of a closed loop current controlled Ćuk converter, the new TS fuzzy modeling approach has been shown to be able to accurately capture all fast-scale nonlinear phenomena occurring at clock frequency with a much reduced level of computational complexity.

A rigorous mathematical analysis based on the piecewise Lyapunov function candidates is introduced to analyze the stability of the limit cycles. All stability conditions have been formulated as Linear Matrix Inequalities (LMI) conditions to predict the onset of the unstable period-doubling border collision bifurcation. Nonsmooth Lyapunov theory, as presented in this paper, is essential to tackle the discontinuous dynamics of the new TS fuzzy model of the converter. The partitioning of fuzzy state-space into relaxed, detached regions reduces the possibility of conservative LMI formulation to a bare minimum. The new approach has been employed to design new switching fuzzy model-based controllers to preserve the stable period-1 behavior of the system for a substantially larger range of parameter variations.

The proposed TS fuzzy modeling approach provides a new method for the analysis of all fast-scale instabilities that may occur in nonsmooth power electronic systems as an alternative to the traditional discrete nonlinear mapping methods usually employed for these systems. It can also be extended to tackle nonsmooth mechanical systems with state jumps.

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